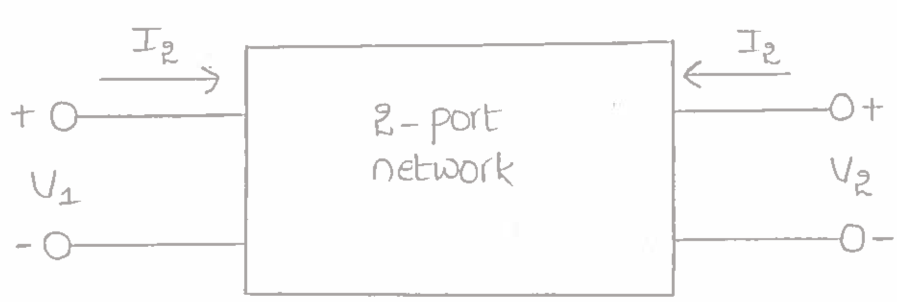
# UNIT - IV

# Scattering matrix

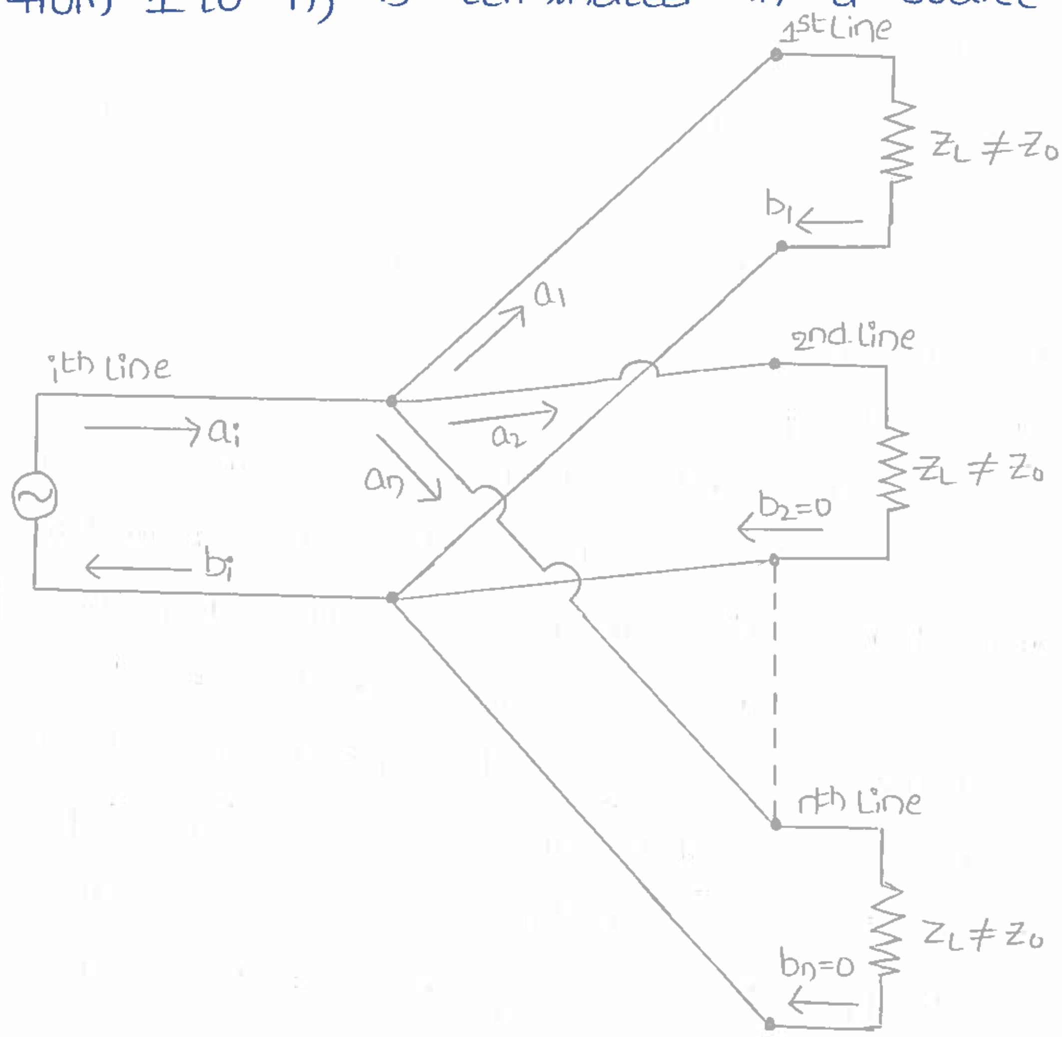
## Scattering Matrix Properties

Low frequency circuits can be described by two port networks and their parameters such as Z, Y, H, ABCD etc. as per network theory. Here network parameters relate the total voltages and total currents.

In a similar way at microwave frequencies, we talk of travelling waves with associated powers instead of voltages and currents and the microwave junction can be defined by what are called as S-parameters or scattering parameters (similar to H, Y, Z parameters). It can be seen that for an input at one port, we have four outputs as discussed earlier. Similarly if we apply inputs to all the ports, We will have 16 combinations, which are represented in a matrix form and that matrix is called as a Scattering Matrix. It is a square matrix which gives all the combinations of power relationships between the various input and output ports of a microwave junction. The elements of this matrix are called scattering coefficients or scattering (S) parameters.



To obtain the relationship between the scattering matrix and the input/output powers at different ports, consider a junction of 'n' number of transmission lines wherein the $i^{th}$ line ($i$ can be any line from 1 to n) is terminated in a source




<u>Case 1</u>: Let the first line be terminated in an impedance other than the characteristic impedance (i.e, $Z_L \neq Z_0$) and all the remaining lines (from 2nd to $n^{th}$ line) in an impedance equal to $Z_0$ (i.e., $Z_L = Z_0$)

If $a_i$ be the incident wave at the junction due to a source at the $i^{th}$ line, then it divides itself among $(n-1)$ number of lines as $a_1, a_2, \ldots a_n$. There will be no reflections from 2nd to $n^{th}$ line and the incident waves are absorbed since their impedances are equal to characteristic impedance ($Z_0$). But, there is a mismatch at the 1st line and hence there will be reflected wave $b_1$ going back into the junction.

$b_1$ is related to $a_1$ by,

$$b_1 = (\text{reflection coefficient})\, a_1 = S_{i1} \cdot a_1$$

where $S_{i1}$ = reflection coefficient of 1st line

1 = reflection from 1st line and

i = source connected at ith line.

Hence, the contribution to the outward travelling wave in the ith line is given by

$$b_i = S_{i1} \cdot a_1 \qquad [\because b_2 = b_3 = \dots = b_n = 0]$$

Case 2: let all the (n-1) lines be terminated in an impedance other than $Z_0$ (i.e., $Z_L \neq Z_0$ for all the lines)

Then, there will be reflections into the junction from every line and hence the total contribution to the outward travelling wave in the ith line is given by

$$b_i = S_{i_1} \cdot a_1 + S_{i_2} \cdot a_2 + S_{i_3} \cdot a_3 + \dots + S_{i_n} \cdot a_n \quad \text{— (1)}$$

$i = 1$ to $n$ since $i$ can be any line from 1 to n

Therefore, we have

$$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + \dots + S_{1n}a_n$$

$$b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + \dots + S_{2n}a_n$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

$$b_n = S_{n1}a_1 + S_{n2}a_2 + S_{n3}a_3 + \dots + S_{nn}a_n$$

In matrix form,

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & & & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \rightarrow (2)$$

column Matrix [b] correspond-ing to Reflected waves or output

Scattering Column Matrix [S] of order n×n

Matrix [a] corresponding to Incident waves or Input

$$\therefore \quad [b] = [S][a] \rightarrow (3)$$

When a junction of n number of waveguides is considered,

a's represent inputs to particular ports

b's represent outputs out of various ports.

$S_{ij}$ corresponds to scattering coefficients resulting due to input at ith port and output taken out of jth port

$S_{ii}$ denotes how much of power is reflected back from the junction into the ith port when input power is applied at the ith port itself.

<u>Properties of [S] Matrix</u>

1. [S] is always as square matrix of order(n×n)
2. [S] is a symmetric matrix

   i.e., $S_{ij} = S_{ji}$
3. [S] is a unitary matrix

   i.e., $[S][S]^* = [I]$

where, $[S]^*$ = complex conjugate of [S]

$[I]$ = unit matrix or Identity matrix of the same order as that of $[S]$.

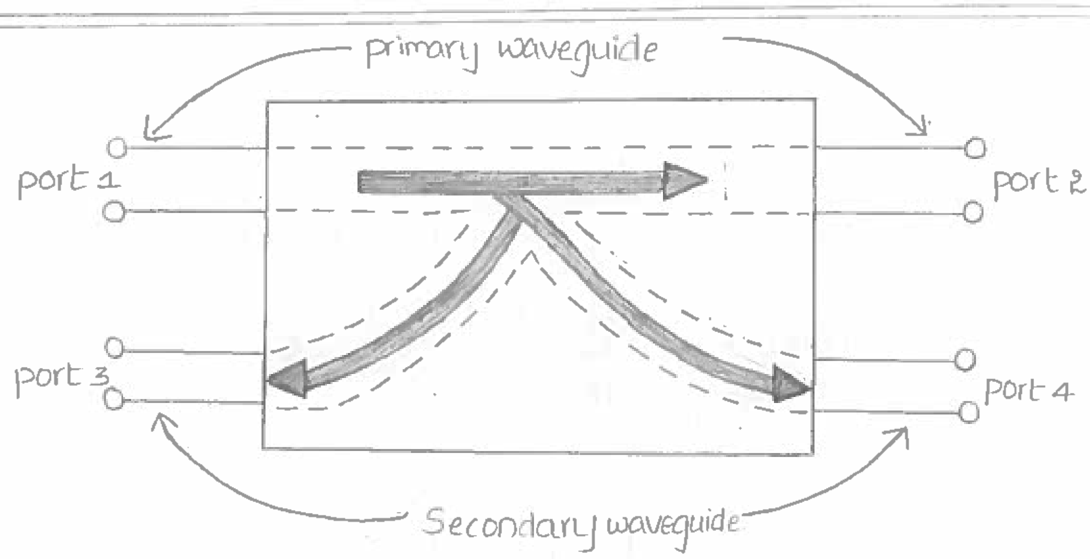
4. The sum of the products of each term of any row (or column) multiplied by the complex conjugate of the corresponding terms of any other row (or column) is zero.

i.e., $\sum_{i=1}^{n} S_{ik} S_{ij}^{*} = 0 \quad k \neq j \quad \begin{bmatrix} k = 1, 2, 3, \ldots n \\ j = 1, 2, 3, \ldots n \end{bmatrix}$

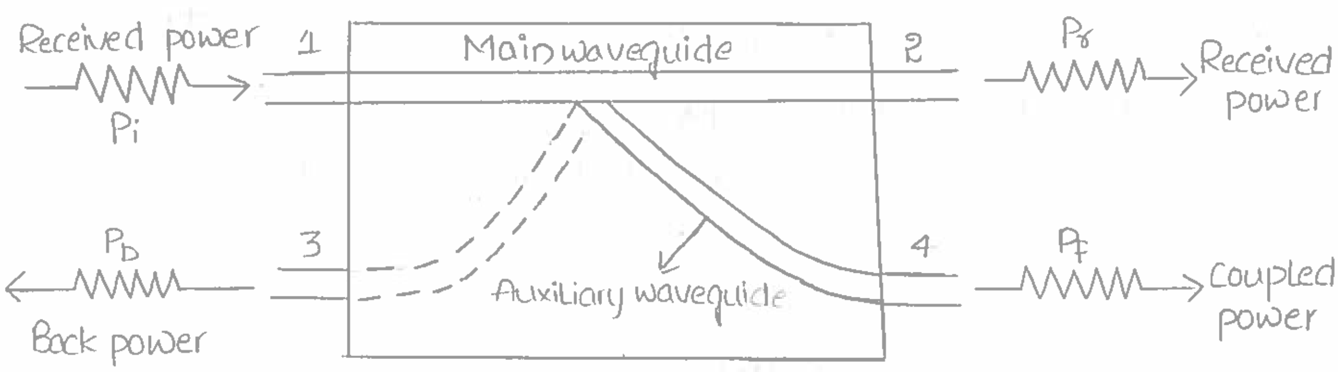
5. If any of the terminal or reference planes (say the $k^{th}$ port) are moved away from the junction by an electric distance $\beta_k l_k$, each of the coefficients $S_{ij}$ involving k will be multiplied by the factor $e^{-j\beta_k l_k}$.

## Directional Couplers - 2 Hole

Directional couplers are flanged, built in waveguide assemblies which can sample a small amount of microwave power for measurement purposes. They can be designed to measure incident and/or reflected powers, SWR (Standing Wave Ratio) values, provide a signal path to a receiver or perform other desirable operations. They can be undirectional (measuring only incident power) or bi-directional (measuring both incident and reflected) powers. In its most common form, the directional coupler is a four port waveguide junction consisting of a primary main waveguide and a auxiliary waveguide as shown in fig.




a) A schematic of a directional coupler




b) Directional coupler indicating powers

With matched terminations at all its ports, the properties of an ideal directional coupler can be summarized as follows.

1. A portion of power travelling from port ① to port ② is coupled to port ④ but not to port ③.
2. A portion of power travelling from port ② to port ① is coupled to port ③ but not to port ④.
3. A portion of power incident on port ③ is coupled to port ② but not to port ① and a portion of the power incident on port ④ is coupled to port ① but not to port ②. Also ports ① and ③ are decoupled as are port ② and ④.

A small portion of input power at port ① is coupled to port ④ so that measurement of this small power is possible. Ideally no power should come out of port ③. The above figure indicates the various input /output powers.

$P_i$ = incident power at port ①

$P_r$ = received power at port ②

$P_f$ = forward coupled power at port ④

$P_b$ = back power at port ③

The performance of a directional coupler is usually defined in terms of two parameters which are defined as follows.

Coupling Factor C: The coupling factor of a directional coupler (D.C) is defined as the ratio of the incident power '$P_i$' to the forward power '$P_f$' measured in dB.

i.e., $$C = 10\log_{10}\frac{P_i}{P_f}\ \text{dB} \quad \rightarrow ①$$

Directivity D: The directivity of a D.C is defined as the ratio of forward power '$P_f$' to the back power '$P_b$' expressed in dB.

i.e., $$D = 10\log_{10}\frac{P_f}{P_b}\ \text{dB} \quad \rightarrow ②$$

For a typical D.C

$$C = 20\text{dB}, \quad D = 60\text{dB}$$

i.e., $$C = 20 = 10\log\frac{P_i}{P_f}$$

$$\frac{P_i}{P_f} = 10^2 = 100$$

or $$P_f = \frac{P_i}{100}$$

Also, $$D = 60 = 10\log\frac{P_f}{P_b}$$

$$\frac{P_f}{P_b} = 10^6$$

or $P_b = \frac{P_f}{10^6} = \frac{P_i}{10^8}$ (since $P_f = \frac{P_i}{100}$)

since $P_b$ is very small, $\left[\frac{1}{10^8}\right] P_i$, the power coming out of port ③ can be neglected.

The Coupling factor is a measure of how much of the incident power is being sampled while directivity is a measure of how well the directional coupler distinguishes between the forward and reverse travelling powers.

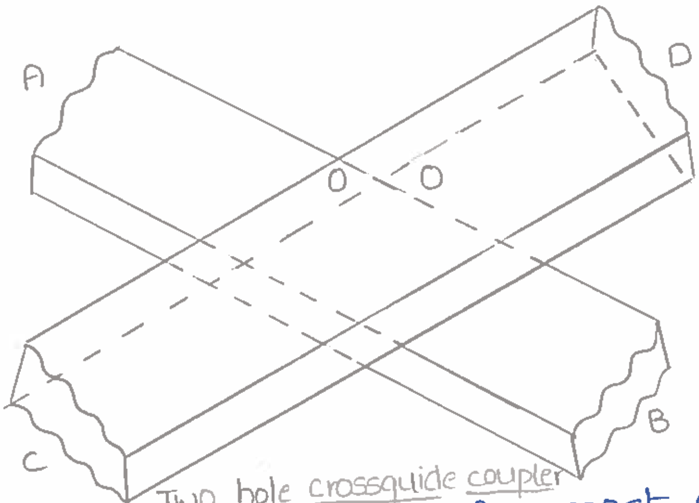
Isolation: Another parameter called Isolation is sometimes defined to describe the directive properties of a directional coupler. It is defined as the ratio of the incident power $P_i$ to the back power $P_b$ expressed in dB

$$I = 10\log_{10}\frac{P_i}{P_b}\ \text{dB} \rightarrow ③$$

It may be noted that isolation in dB equals coupling factor plus directivity.

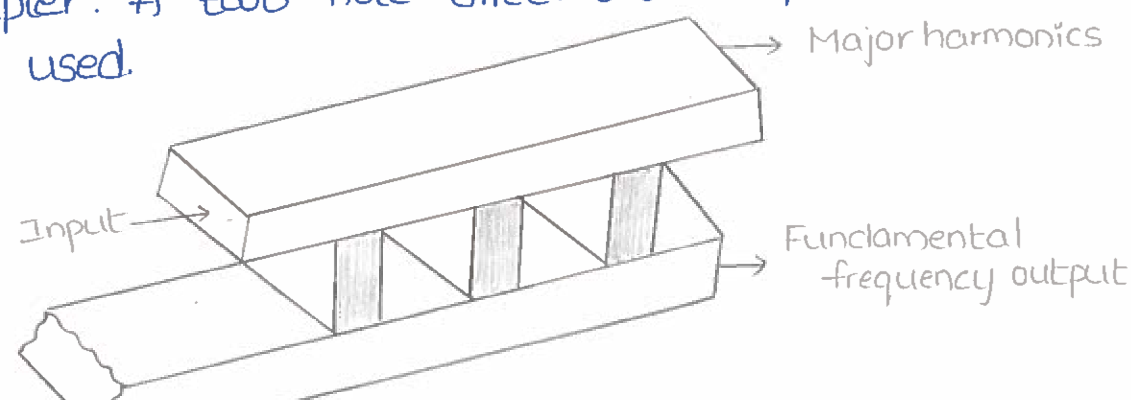
In addition to the above parameters the SWR, frequency range and transmission loss are also specified for a directional coupler. Low SWR ensures minimum mismatch errors, wide frequency range eliminates the need for several octave band couplers to cover the broad band range and minimum transmission loss for significant power availability for measurement set up.

There are several types of directional couplers that have been developed like Two hole crossed guide couplers with common broad wall-sections, branching guide couplers with a common wall instead of coupling holes, short slot couplers, bifurcated couplers, loop directional coupler, couplers made from parallel ground plane, metallic strips running internally within the waveguide structure.

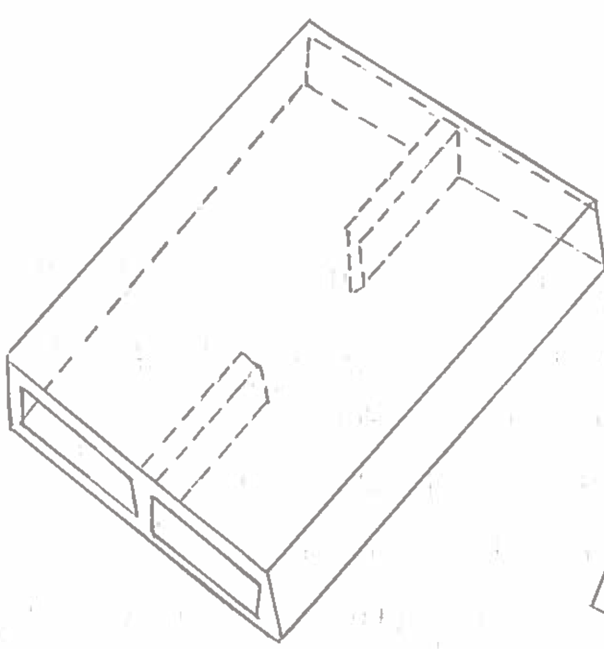



Two hole crossguide coupler

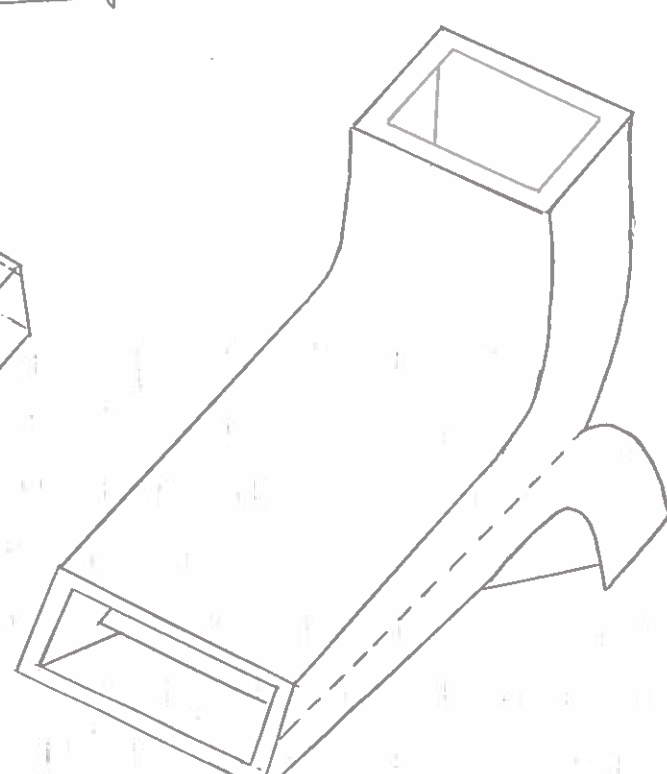
It may be noted that in most of the directional couplers only three of the four ports are used, the unwanted port is normally terminated in a matched load built into it. The two waveguides share a common wall. This common wall has got gole or holes for coupling the energy flowing into the main waveguide to the side waveguide and hence called a side hole coupler. A two hole directional coupler is most commonly used.



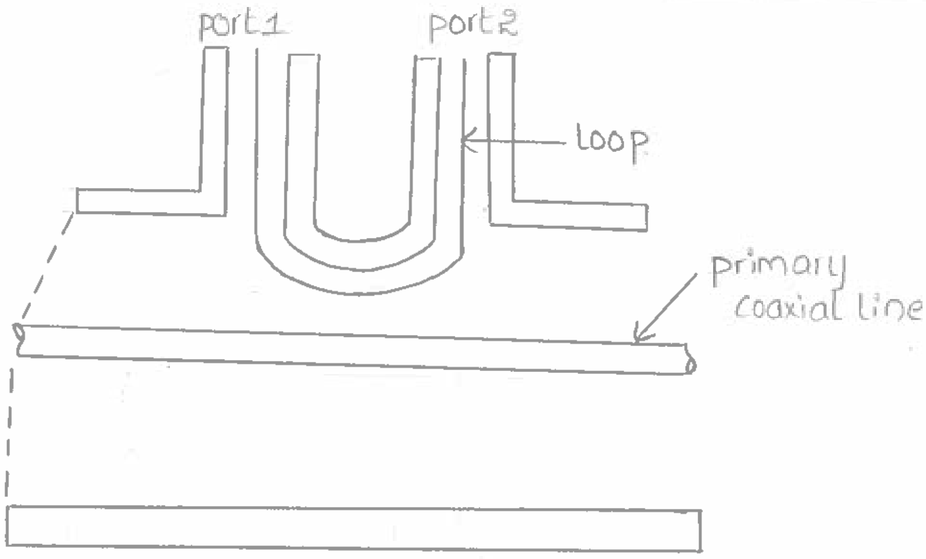

Two hole branching guide coupler



Short-slot coupler



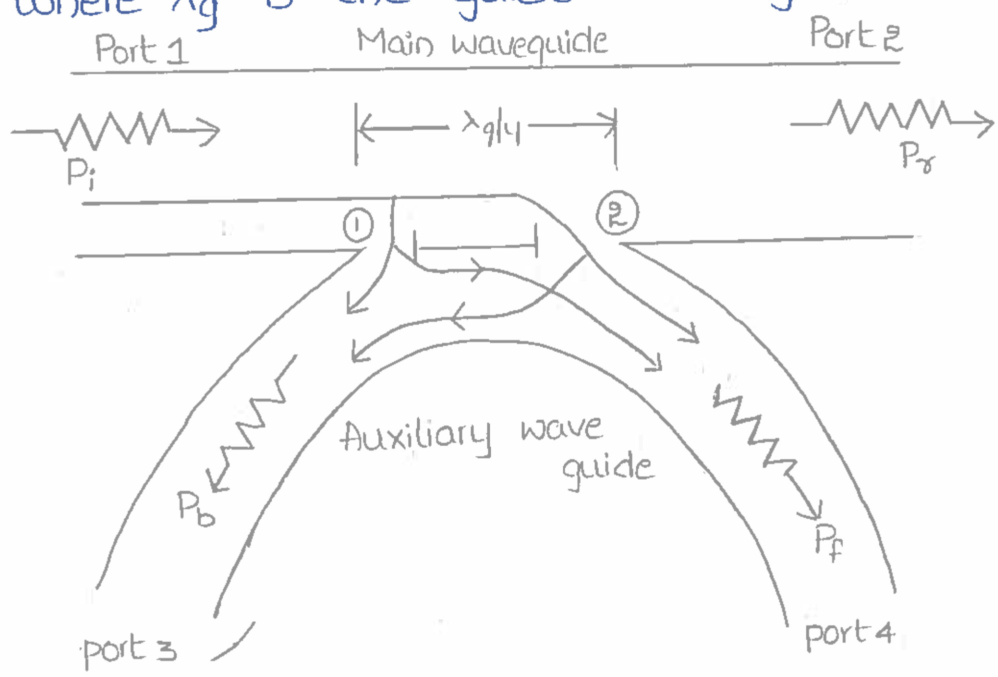
Bifurcated coupler




Loop directional coupler

## Two-hole Directional Coupler

The principle of operation of a two-hole directional coupler. It consists of two guides the main and the auxiliary with two tiny holes common between them as shown. The two holes are at a distance of $\lambda_g/4$ where $\lambda_g$ is the guide wavelength.




Two hole directional coupler

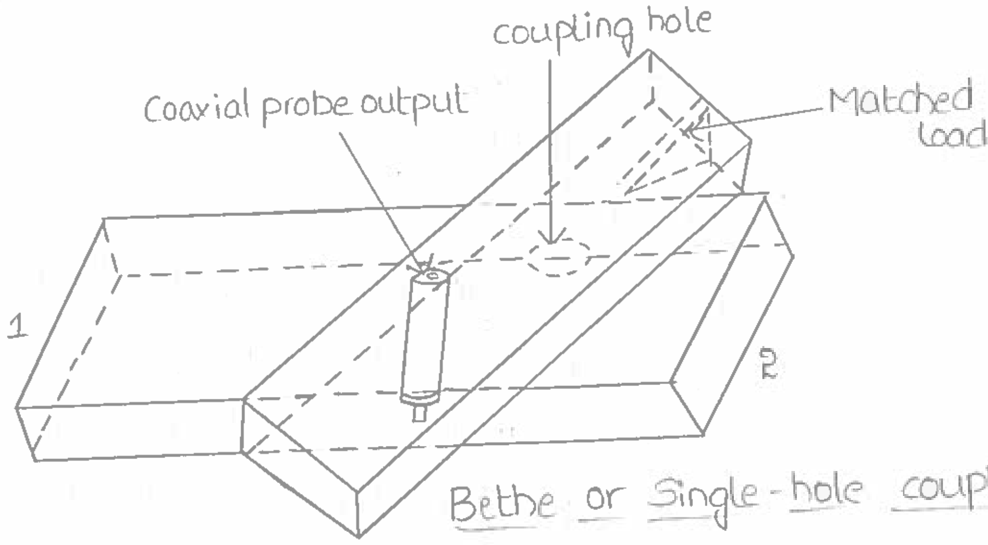
The two leakages of holes ① and ② both in phase at the position of 2nd hole and hence they add up contributing to $P_f$. But the two leakages are out of phase by 180° at the position of the 1st hole and therefore they cancel each other making $P_b = 0$. The magnitude of the power coming out of 2 holes depend upon the dimension of the two holes. Since the distance between holes is $\lambda g/4$, $P_b$ is made '0' (since the incident power will have to travel a distance of $(\lambda g/4 + \lambda g/4)$ when it comes back from hole ② resulting in 180° phase shift. Compared to incident power leakage through hole ① entering port ③.

The number of holes can be one (as in Bethe crossguide coupler) or more than two (as in a Multi-hole coupler). The degree of coupling is determined by size and location of the holes in the waveguide walls.

Although a high degree of directivity can be achieved as a fixed frequency, it is quite difficult over a band of frequencies. In this connection, it should be realized that the frequency determines the seperation of the two holes as a fraction of the wavelength.

## Bethe hole

A single-hole directional coupler is shown. Here the directivity is improved as the Bethe coupler relies on a single hole for coupling process rather than the seperation between two holes. The power entering port① is coupled to the co-axial probe output and the power entering port ② in absorbed by the matched load. The auxiliary guide is placed at such an angle that the magnitude of the magnetically excited wave a made equal to that of the electric-ally excited wave for improved directivity. In this coupler, the waves in the auxiliary guide are generat-ed through a single hole which includes both electric and magnetic fields. Because of the phase relationships involved in the coupling process, the signals generated by the two types of coupling cancel in the forward direction and reinforce in the reverse direction.




Bethe or Single-hole coupler

## Scattering Matrix of a Directional Coupler

We use the properties of the directional coupler to arrive at the [S] matrix.

1. Directional coupler is a four port network. Hence [S] is a 4x4 matrix.

i.e.,
$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \rightarrow (1)$$

2. In a directional coupler all four ports are perfectly matched to the junction. Hence the diagonal elements are zero.

i.e., $$S_{11} = S_{22} = S_{33} = S_{44} = 0 \rightarrow (2)$$

3. From symmetric property, $S_{ij} = S_{ji}$

$$S_{23} = S_{32};\ S_{13} = S_{31};\ S_{24} = S_{42};$$
$$S_{34} = S_{43};\ S_{41} = S_{14} \rightarrow (3)$$

Ideally back power is zero ($P_b = 0$) i.e., There is no coupling between port (1) and port (2)

$$\therefore\ S_{13} = S_{31} = 0 \rightarrow (4)$$

4. Also there is no coupling between port (2) and port (4)

$$S_{24} = S_{42} = 0 \rightarrow (5)$$

Substituting the values of scattering parameters, we get

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \longrightarrow (6)$$

5. Since $[S][S^*] = [I]$, we get

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow (7)$$

$R_1C_1 = |S_{12}|^2 + |S_{14}|^2 = 1 \longrightarrow (7)$

$R_2C_2 = |S_{12}|^2 + |S_{23}|^2 = 1 \longrightarrow (8)$

$R_3C_3 = |S_{23}|^2 + |S_{34}|^2 = 1 \longrightarrow (9)$

$R_1C_3 = S_{12}S_{23}^* + S_{14}^*S_{34}^* = 0 \longrightarrow (10)$

Comparing eq (2), (3)

$$S_{14} = S_{23} \longrightarrow (11)$$

Comparing eq (3) & (4),

$$S_{12} = S_{34} \longrightarrow (12)$$

Let us assume that $S_{12}$ is real and positive = 'P'

$$\therefore \quad S_{12} = S_{34} = P = S_{34}^* \longrightarrow (13)$$

From eqn (10) to (13),

$$PS_{23}^* + S_{23}P = 0$$

$$\therefore \quad P[S_{23}^* + S_{23}] = 0$$

Since, $P \neq 0$, $S_{23} + S_{23}^* = 0$

$$S_{23} = jy$$

$$S_{23}^* = -jy$$

i.e., $S_{23}$ must be imaginary.

let $S_{23} = jq = S_{14}$ → (14)

Therefore,

$S_{12} = S_{34} = P$ (transmission parameter)

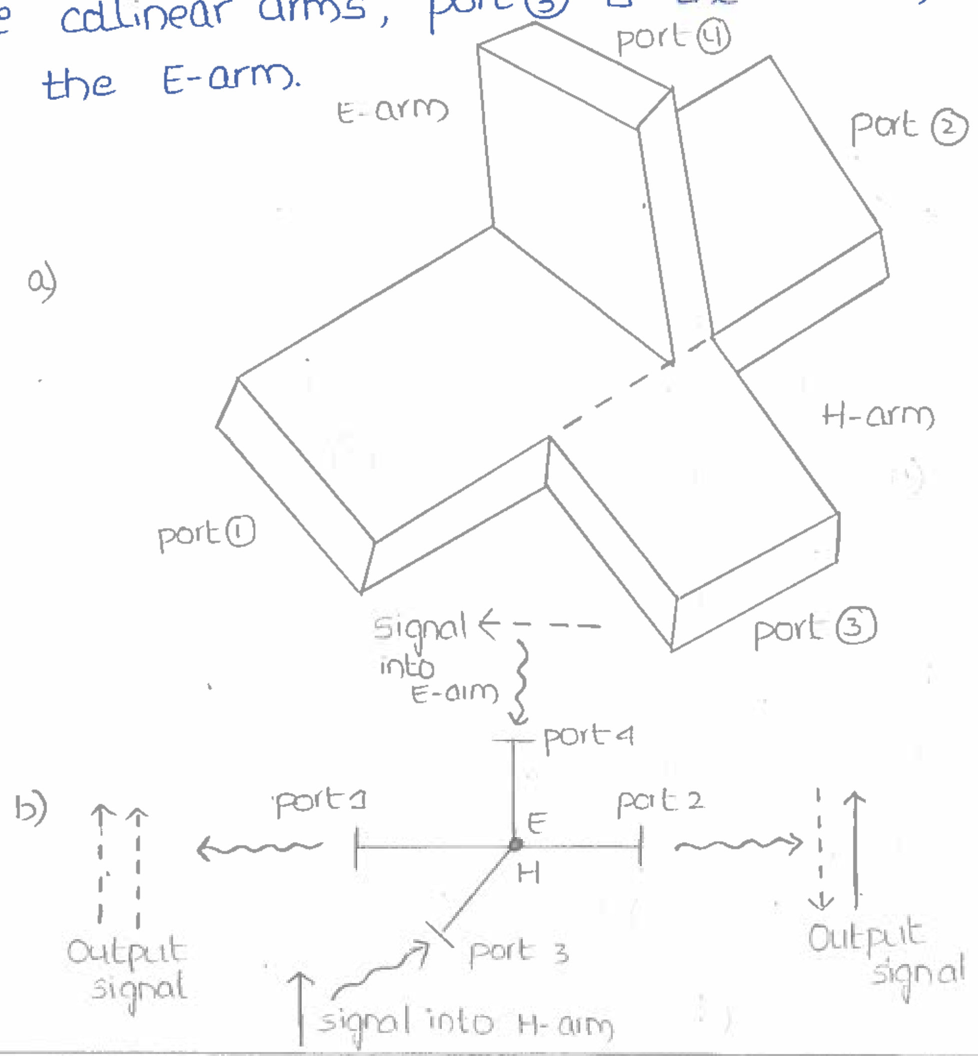
$S_{23} = S_{14} = jq$, Also, $p^2 + q^2 = 1$

and substituting these values in eq (6), [S] matrix of a directional coupler is reduced to

$$[S] = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix} \rightarrow (15)$$

## [S] Matrix of Magic Tee

Here rectangular slots are cut both along the width and breadth of a long waveguide and side-arms are attached as shown in fig. Port (1) and (2) are collinear arms, port (3) is the H-arm, and port (4) is the E-arm.



Such a device became necessary because of the difficulty of obtaining a completely matched three port Tee junction. This four port hybrid Tee junction combines the power dividing properties of both H-plane Tee and E-plane Tee. and has the advantage of being completely matched at all its ports. This has several useful applications as will be seen later. Using the properties of E-H plane Tee, its scattering matrix can be obtained as follows.

1. $[S]$ is a 4x4 matrix since there are 4 ports.

i.e.,
$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \text{--- (1)}$$

2. Because of H-plane Tee section

$$S_{23} = S_{13} \quad \rightarrow (2)$$

3. Because of E-plane Tee section

$$S_{24} = -S_{14} \quad \rightarrow (3)$$

4. Because of geometry of the junction an input at port (3) cannot come out of port (4) since they are isolated ports and vice versa.

$$\therefore \; S_{34} = S_{43} = 0 \quad \rightarrow (4)$$

5. From symmetric property, $S_{ij} = S_{ji}$

$$S_{12} = S_{21} \; ; \; S_{13} = S_{31} \; ; \; S_{23} = S_{32} \; ;$$
$$S_{34} = S_{43} \; ; \; S_{24} = S_{42} \; ; \; S_{41} = S_{14} \; ; \quad \rightarrow (5)$$

6. If ports (3) and (4) are perfectly matched to the junction

$$S_{33} = S_{44} = 0 \quad \rightarrow (6)$$

Substituting the above properties from eq ② to ⑥ in ①, we get

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \rightarrow ⑦$$

7. From unitary property, $[S][S]^* = [I]$

i.e., $$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1C_1 = |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \rightarrow ⑧$$

$$R_2C_2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \rightarrow ⑨$$

$$R_3C_3 = |S_{13}|^2 + |S_{13}|^2 = 1 \rightarrow ⑩$$

$$R_4C_4 = |S_{14}|^2 + |S_{14}|^2 = 1 \rightarrow ⑪$$

From eq ⑩ & ⑪, we get

$$S_{13} = \frac{1}{\sqrt{2}} \rightarrow ⑫$$

$$S_{14} = \frac{1}{\sqrt{2}} \rightarrow ⑬$$

Comparing eqn ⑧ and ⑨, we get

$$S_{11} = S_{22} \rightarrow ⑭$$

Using these values from eqns ⑫ and ⑬ in eq ⑬,

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$\therefore |S_{11}|^2 + |S_{12}|^2 = 0$$

i.e., $$S_{11} = S_{12} = 0 \rightarrow ⑮$$

∴ From eq ⑨, $$S_{22} = 0 \rightarrow ⑯$$

This means port ① and ② are also perfectly matched to the junction. Hence in any four port junction, if any two ports are perfectly matched to the junction, then the remaining two ports are automatically matched to the junction. Such a junction where in all the four ports are perfectly matched to the junction is called a Magic Tee.

The [S] of Magic Tee is obtained by substit-uting the scattering parameters from eq (12) to (16) in eq (7)

$$[S] = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix} \rightarrow (17)$$

We know that, $[b] = [S][a]$

i.e.,

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -1/\sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$\therefore$

$$\left.\begin{aligned} b_1 &= \frac{1}{\sqrt{2}}(a_3+a_4) \\ b_2 &= \frac{1}{\sqrt{2}}(a_3-a_4) \\ b_3 &= \frac{1}{\sqrt{2}}(a_1+a_2) \\ b_4 &= \frac{1}{\sqrt{2}}(a_1-a_2) \end{aligned}\right\} \rightarrow (18)$$

Using Eq (18), we look at the properties of Magic Tee for some important cases.

Case 1: $a_3 \neq 0,\ a_1 = a_2 = a_4 = 0$

Substituting these in eq (18), we get

$$b_1 = \frac{a_3}{\sqrt{2}}\ ;\ b_2 = \frac{a_3}{\sqrt{2}}\ ;\ b_3 = b_4 = 0$$

This is the property of H-plane Tee.

**Case 2:** $a_4 \neq 0,\ a_1 = a_2 = a_3 = 0$

$$\therefore \quad b_1 = \frac{a_4}{\sqrt{2}}\ ;\ b_2 = \frac{-a_4}{\sqrt{2}}\ ;\ b_3 = b_4 = 0$$

This is the property of E-plane Tee

**Case 3:** $a_1 \neq 0,\ a_2 = a_3 = a_4 = 0$

$$\therefore \quad b_1 = 0\ ;\ b_2 = 0;\ b_3 = \frac{a_1}{\sqrt{2}}\ ;\ b_4 = \frac{a_1}{\sqrt{2}}$$

i.e., when power is fed into port ①, nothing comes out of port ② even though they are collinear ports (Magic!!) Hence ports ① and ② are called isolated ports. Similarly an input at port ② cannot come out at port ①. Similarly E and H are isolated ports.

**Case 4:** $a_3 = a_4,\ a_1 = a_2 = 0$

Then $$b_1 = \frac{1}{\sqrt{2}}(2a_3)\ ;\ b_2 = 0,\ b_3 = b_4 = 0$$

This is nothing but the additive property. Equal inputs at port ③ and ④ result in an output at port ① (in phase and equal in amplitude)

**Case 5:** $a_1 = a_2,\ a_3 = a_4 = 0;$

$$\therefore \quad b_1 = 0 = b_2 = b_4;\ b_3 = \frac{1}{\sqrt{2}}(2a_1)$$

that is equal inputs at port ① and ② results in an output at port ③ (additive property) and no outputs at ports ①, ② and ④. This is similar to case 4.

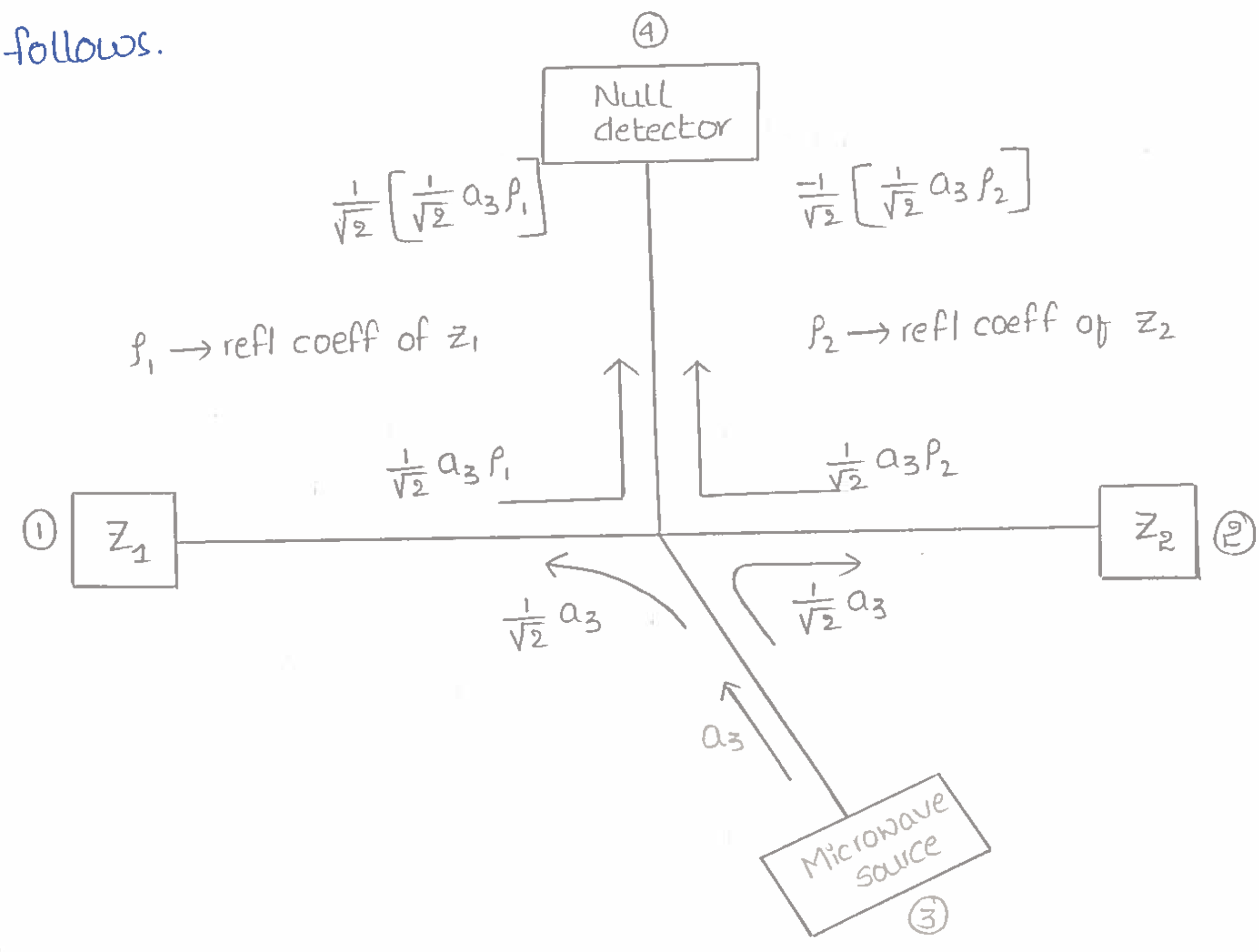
**Applications of Magic Tee:**

A magic Tee has several applications. A few of them have been discussed here.

## a) Measurement of Impedance:

A magic Tee has been used in the form of a bridge for measuring impedance.

Microwave source is connected in arm ③ and a null detector in arm ④. The unknown impedance is connected in arm ② and a standard variable known impedance in arm ①. Using the properties of Magic Tee, the power from microwave source ($a_3$) gets divided equally between arms ① and ② $\left[\frac{a_3}{\sqrt{2}}\right]$. These impedances are not equal to characteristic impedance $Z_0$ and hence there will be reflections from arm ① and ②. If $\rho_1$ and $\rho_2$ are the reflection coefficient, powers $\frac{\rho_1 a_3}{\sqrt{2}}$ and $\frac{\rho_2 a_3}{\sqrt{2}}$ enter the Magic Tee junction from arm ① and ② as shown in fig. The resultant wave into arm ④ i.e., the null detector can be calculated as follows.




Magic Tee for measurement of impedances

The net wave reaching the null detector

$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}} a_3 \rho_1\right] - \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}} a_3 \rho_2\right] = \frac{1}{2} a_3(\rho_1 - \rho_2) \rightarrow ①$$

For perfect balancing of the bridge (null detection) eq ① is equated to zero.

$$\frac{1}{2} a_3 (\rho_1 - \rho_2) = 0$$

$$\rho_1 - \rho_2 = 0 \quad (\text{or}) \quad \rho_1 = \rho_2$$

(or)
$$\frac{Z_1 - Z_Z}{Z_1 + Z_Z} = \frac{Z_2 - Z_Z}{Z_2 + Z_Z}$$

$$\therefore \quad Z_1 = Z_2$$

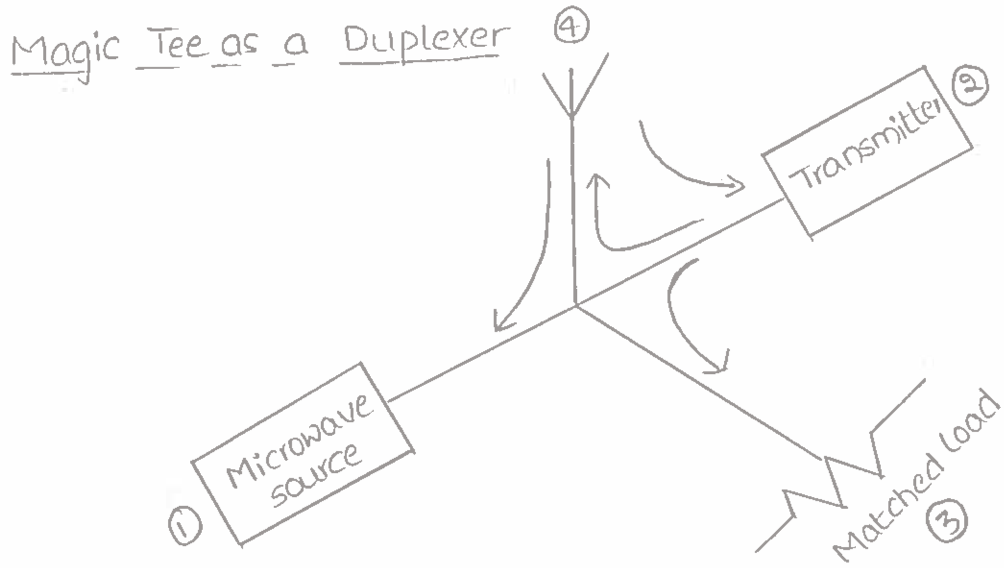
i.e.,
$$R_1 + jX_1 = R_2 + jX_2$$

$$R_1 = R_2 \quad \text{and} \quad X_1 = X_2.$$

Thus the unknown impedance can be measured by adjusting the standard variable impedance till the bridge is balanced and both impedance become equal.

b) <u>Magic Tee as a Duplexer:</u>

The transmitter and receiver are connected in port ② and ① respectively, antenna in the E-arm or port ④ and port ③ of Magic Tee is terminated in a matched load. During transmission half the power reaches the antenna from where it is radiated into space. Other half reaches the matched load where it is absorbed without reflections. No transmitter power reaches the receiver since port ① and ② are isolated ports in a Magic Tee. During reception, half of the received power goes to the receiver and the other half to the transmitter are isolated during reception as well as during transmission.



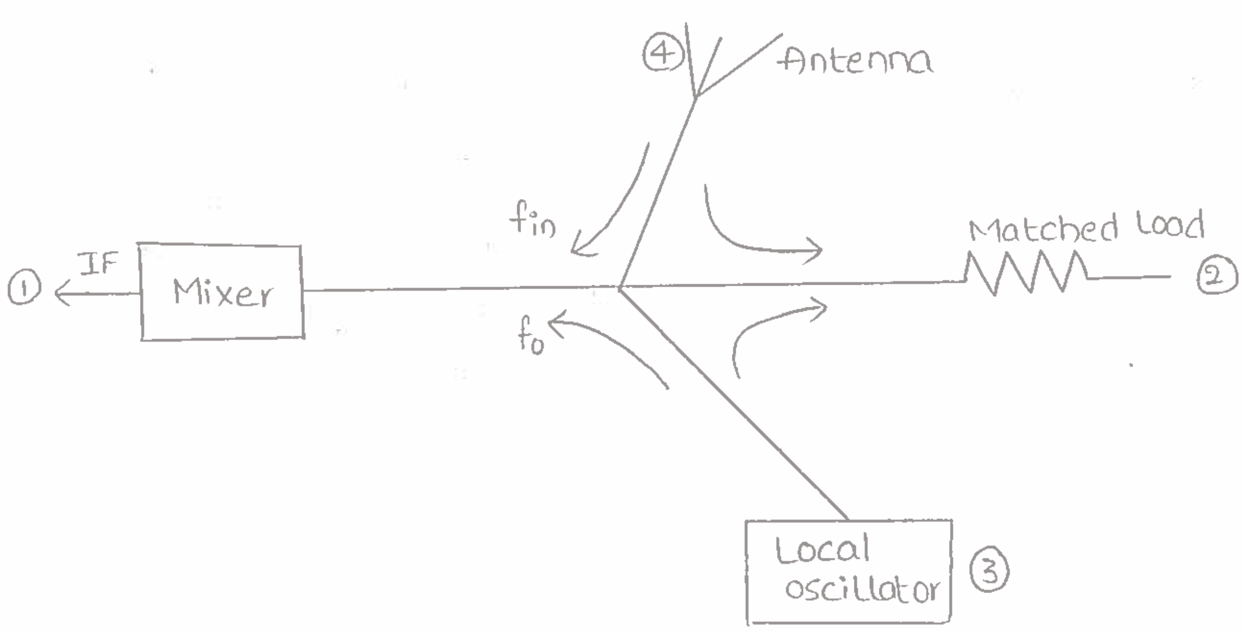

c) Magic Tee as a Mixer:

A magic Tee can also be used in microwave receivers as a mixer where the signal and local oscillator are fed into the E and H arms.

Half of the local oscillator power and half of the received power from antenna goes to the mixer where they are mixed to generate the IF frequency

$$IF = f_{in} \sim f_o$$

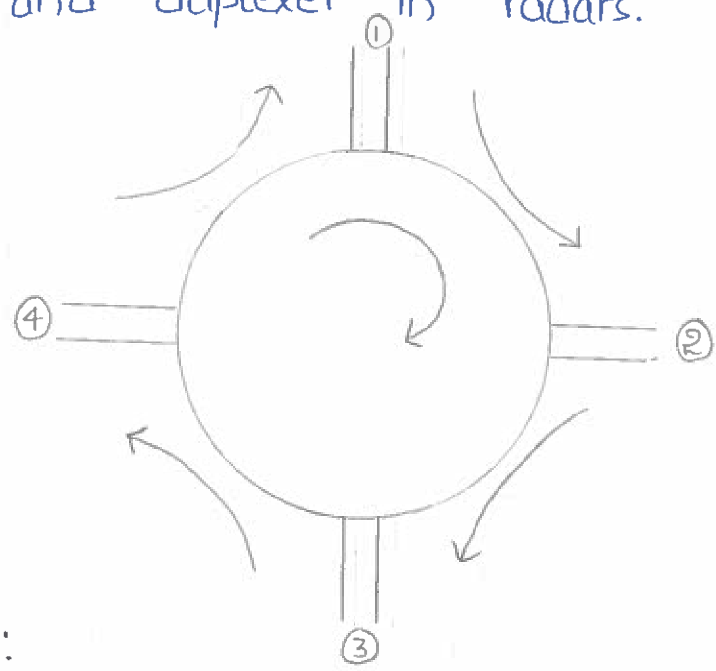
Magic Tee has many other applications such as a microwave discriminator, microwave bridge etc




Magic Tee as a mixer

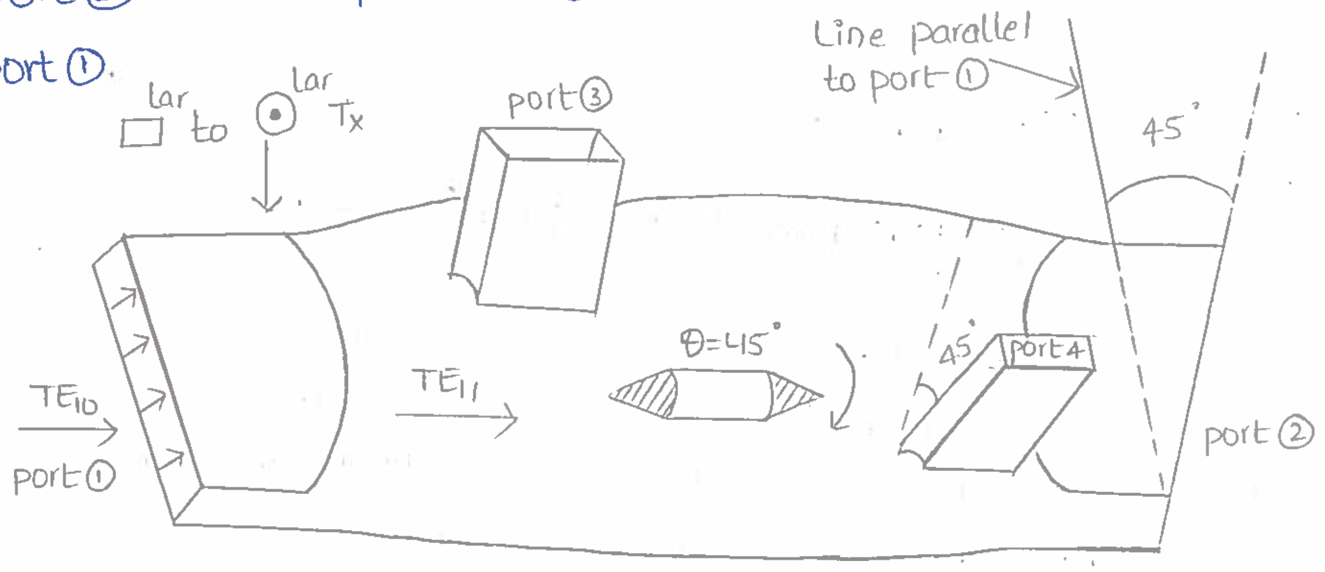
## Circulator

A circulator is a four port microwave device which has a peculiar property that each terminal is connected only to the next clockwise terminal i.e., port ① is connected to port ② only and not to port ③ and ④ and port ② is connected only to port ③ etc. Although there is no restriction on the number of ports, four ports are most commonly used. They are useful in parametric amplifiers, tunnel diode, amplifiers and duplexer in radars.




## Construction:

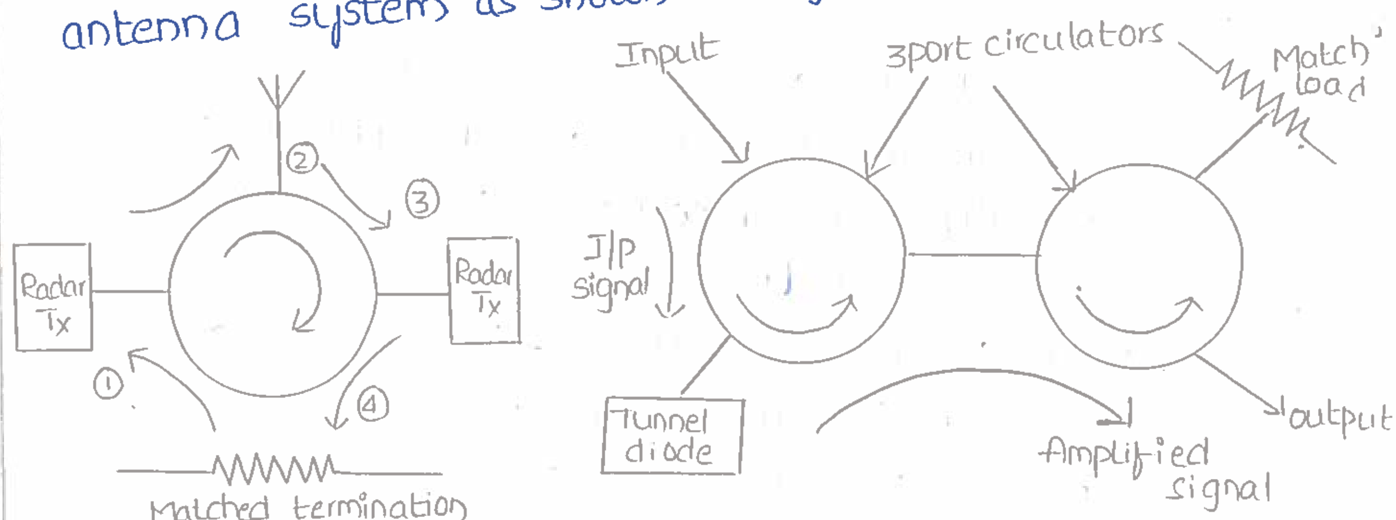
A four port Faraday rotation circulator is shown. The power entering port ① is $TE_{10}$ mode and is converted to $TE_{11}$ mode because of gradual rectangular to circular transition. This power passes port ③ unaffected since the electric field is not significantly cut and is rotated through 45° due to the ferrite, passes port ④ unaffected and finally emerges out of port ②. Power from port ② will have plane of polarization already tilted by 45° with respect to port ①. This power passes port ④ unaffected because again the electric field is not significantly cut. This wave gets rotated by another 45° due to ferrite rod in the clockwise direction. This power whose plane of polarization is tilted through 90° finds port ③ suitably aligned and emerges out of it. Similarly port ③ in coupled only to port ④ and port ④ to port ①.




Four port circulator

Applications:

1. A circulator can be used as a duplexer for a radar antenna system as shown in fig a)




Transmitter feeds the antenna while the received energy is directed to the receiver. The powerful radar transmitter is isolated from the sensitive receiver and also the same antenna can be used for both transmission and reception. This is the duplexer action being performed by a circulator.

2. We can have three port circulators, strip line circulat-ors that can have several applications. Two three

port circulators can be used in tunnel diode or parametric amplifiers as shown.

3. Circulators can be used as low power devices as they can handle low powers only.

## Microwave Measurements

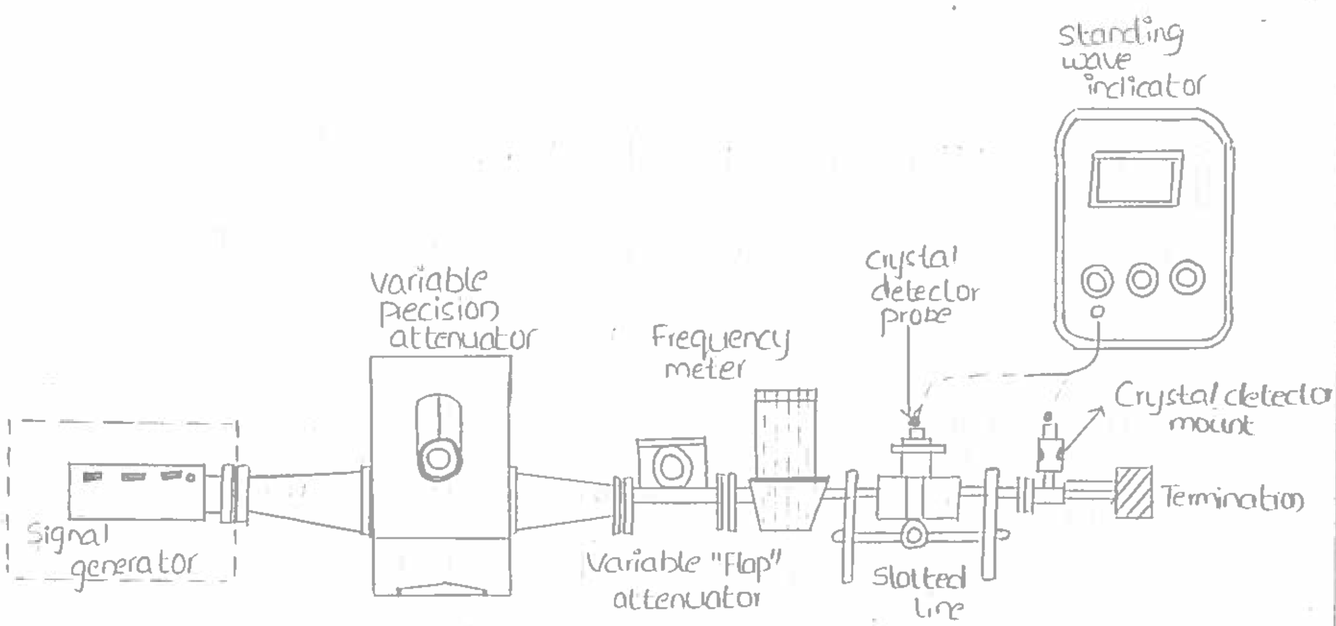
Low Frequency Measurement vs. Microwave Measurements

- At low frequency, it is convenient to measure voltage and current and use them to calculate power. However at microwave frequencies, they are difficult to measure and since they vary with position in a transmission line, are of little value in determining power. Therefore at microwave frequencies, it is more desirable and simpler to measure power directly.
- At low frequency, circuits use lumped elements which can be identified and measured. At microwave frequencies circuit elements are distributed and as such it usually not important to know what element make up a line. It is possible and also satisfactory to measure the impedance of a circuit without regard to the individual distributed elements making up that circuit.
- Unlike low frequency measurements, many quantities measured at microwave frequencies (or their difference in dBs) rather than exact input or output powers.

The following parameters can be conveniently measured at microwave frequencies 1) Frequency 2) Power 3) Attenuation 4) Voltage standing wave Ratio (VSWR) 5) Phase 6) Impedance 7) Insertion loss 8) Dielectric constant 9) Noise factor

## Description of Microwave Bench- Different Blocks

The general setup for measurement of any param -eter in microwaves is normally done by a microwave bench. The signal generator is a microwave source whose output is of the order of milliwatts. It could be a Gunn diode oscillator, a backward wave oscillator or a reflex klystron tube. It can provide either a continuous wave (CW) or square wave modulated at an audio rate which is normally 1kHZ. In some cases, it may have provision for sweep oscillator which allows the output cases it may have provision for sweep oscill -ator which allows the output frequency to be varied periodically. The precision attenuator can provide 0 to 50dB attenuation above its insertion loss. The variable flat attenuator is also used in addition, whose calibr- ation can be checked against readings of the precision attenuator. A frequency meter is used for direct reading of frequency that consists of single cylindrical cavity which can be adjusted to resonance and is slot coupled to the waveguide. The slotted line carriage has just been described. The crystal detector, inserted in the E probe of the slotted line is contained in the crystal detector mount at the end of the waveguide run is used to detect the modula -ted signal. the SWR indicator is basically a sensitive tuned voltmeter that provides direct reading of the SWR or its equivalent value in decibels.




General set-up of microwave bench

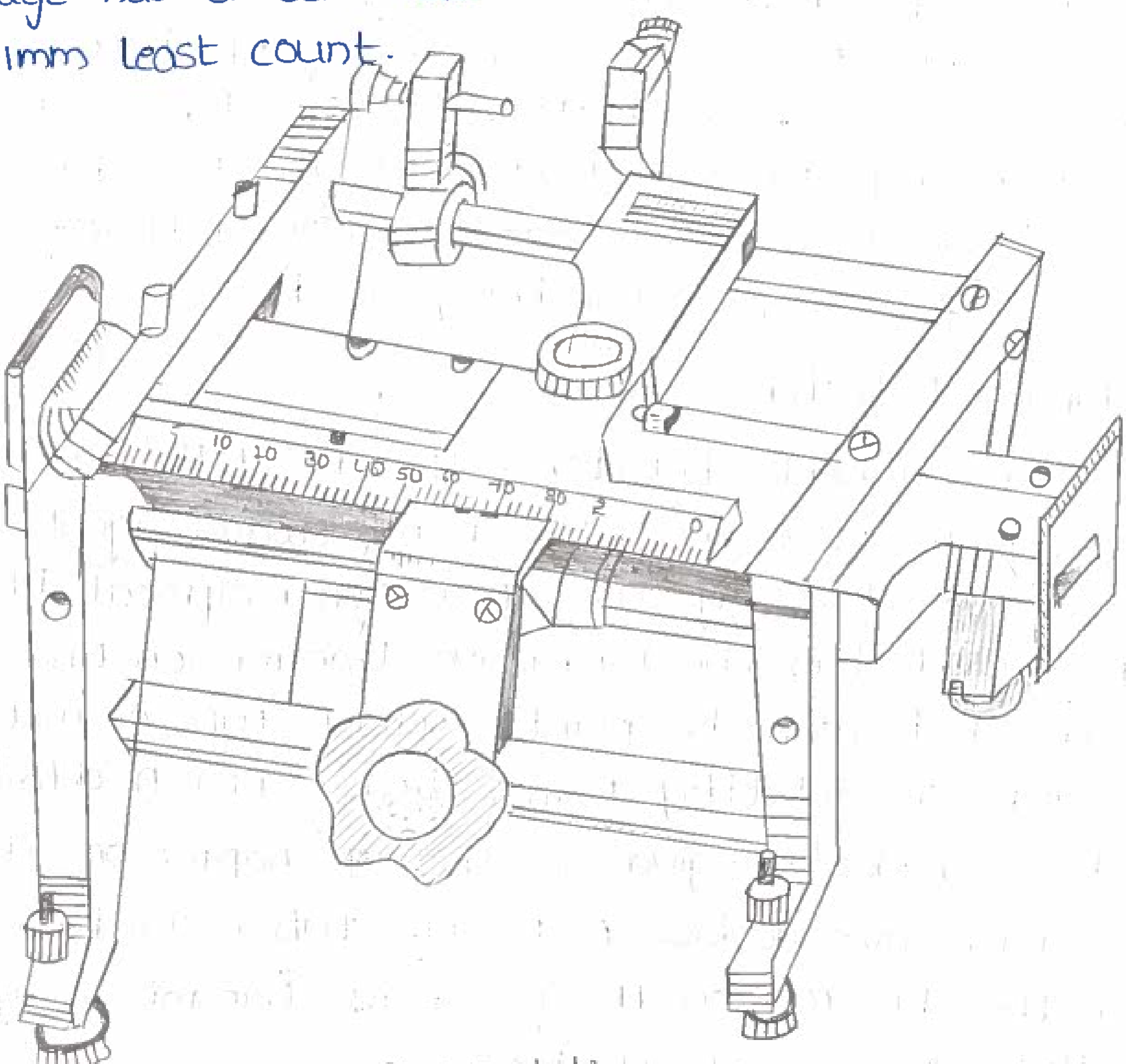
## Measurement devices and Instrumentation

Before different parameters can be measured it shall be appropriate if the several devices and instrumentation that are normally used are discussed. Here, we give a brief descriptions of some of the devices and instruments such as Slotted line carriage, Tunable detector, VSWR meter, Power meter, Wave meter, Spectrum analyser, Network analyser etc.

### i) Slotted Line:

A coupling probe moving along the waveguide can be used to detect the standing wave pattern present inside the waveguide. It is basically used for measuring the standing wave ratio. It consists of a slotted section of a transmission line a travelling probe carriage and facility for attaching detecting instruments. The slot is made in the centre of the broad face of the waveguide parallel to the axis of the waveguide. For the dominant mode travelling inside the waveguide, the slot does not radiate any power.

A small probe inserted through the slot senses the relative field strength of the standing wave pattern inside the waveguide. This probe is on a carriage plate which moves on the top surface of the waveguide. The probe is connected to a crystal detector so that the output from the detector is proportional to the square of the input voltage at that position of the probe. As the position of the probe is moved along the waveguide slot, it gives an output proportional to the standing wave pattern inside the waveguide. Since the crystal is a square law device, the square root of the ratio of maximum output to the minimum output when the probe carriage or travelling probe is moved along the slot gives the VSWR. For noting the positions, the precision built probe carriage has a centimeter scale with a vernier reading of 0.1mm least count.



Slotted line

Thus the slotted line carriage with a tunable detector can be employed to obtain the low frequency modulating signal on an oscilloscope

The probe extends into the slot coming quite close to the inner conductor of the line but not touching it. This loose coupling between line and probe is adequate for measurement purposes. The slotted line will have same characteristic impedance as the main line to which it is connected in series It has a length slightly greater than half wavelength at the lowest frequency of operation. It permits convenient and accurate measurement of the position and size of the first voltage maximum from the load and any subseq-uent ones without significantly interfering with the quantities being measured.

Further it is necessary that the probe be quite thin as compared to the wavelengths and the depth also be small enough to avoid any field distortion. The slotted line carriage with tunable detector can be used to measure impedance, reflection coefficient and return loss in addition to SWR, standing wave pattern and frequency of the generator being used.

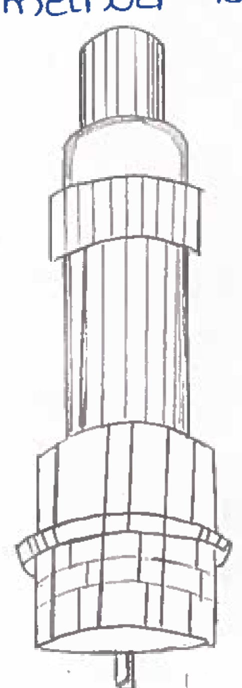
ii) Tunable Detector

The Tunable Detector helps detect the low freque-ncy square wave modulated microwave signal. That is made possible by use of a non-reciprocal detector diode mounted in the microwave transmission line. The detector diode can be point contact type or metal-semiconductor Schottky Barrier diode (SBD). A details of these diodes is given in the chapter on solid state microwave devices. A tunable stub is used to match the detector on the microwave transmission system and there are 3 different types.

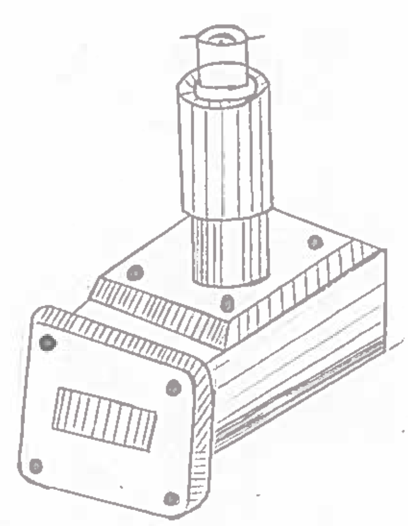
1. Tunable waveguide detector
2. Tunable co-axial detector and
3. Tunable probe detector.

iii) VSWR Meter:

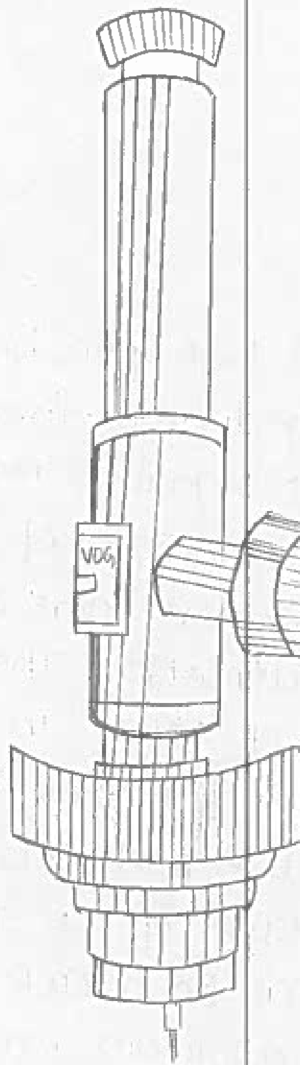
A VSWR meter basically consists of a high gain, high Q,
low noise voltage amplifier normally tuned at a fixed
frequency (1kHz) at which the microwave signal is modula
-ted. The VSWR meter uses the detected signal out of the
microwave detector as its input, amplifies the same and
provides the output on a calibrated voltmeter. The
meter itself can be calibrated in terms of VSWR. In this
case, the probe carriage is moved to give maximum deflect
-ion on the VSWR meter by adjusting the pad. This full
scale deflection (FSD) corresponds to a VSWR of 1 as shown
As an example, an FSD of 10mV corresponds to a VSWR of 1.
The travelling probe is adjusted to get minimum reading
on the meter. If this corresponds to 5mV, then
$VSWR = \frac{10mV}{5mV} = 2$. If it is 3.3mV, VSWR = 3, if it is 2.5mV,
VSWR = 4. If it is 1mV, VSWR = 10 etc, i.e. Such a calibrated
VSWR meter gives an expanded scale upto a VSWR of 2
but for VSWR >10 the meter will be congested and the
measurement will not be accurate for VSWR's >10. Hence
this method is not useful for VSWR's >10.



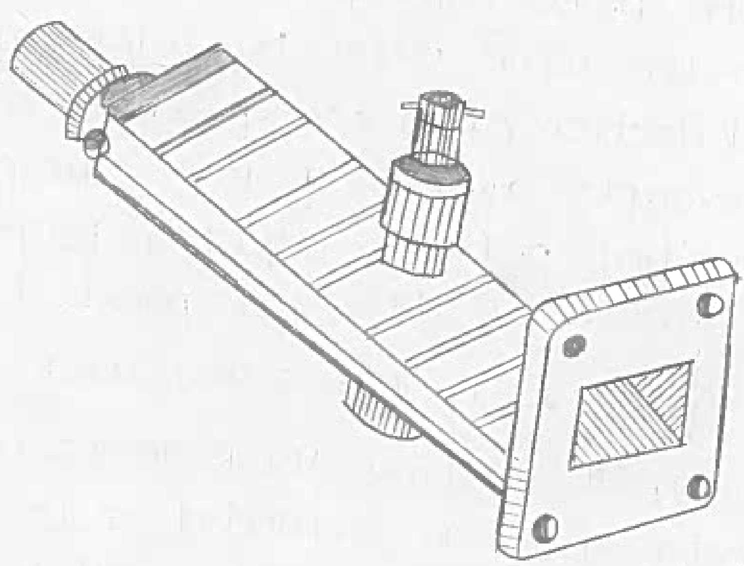
Fixed board band tuned probe



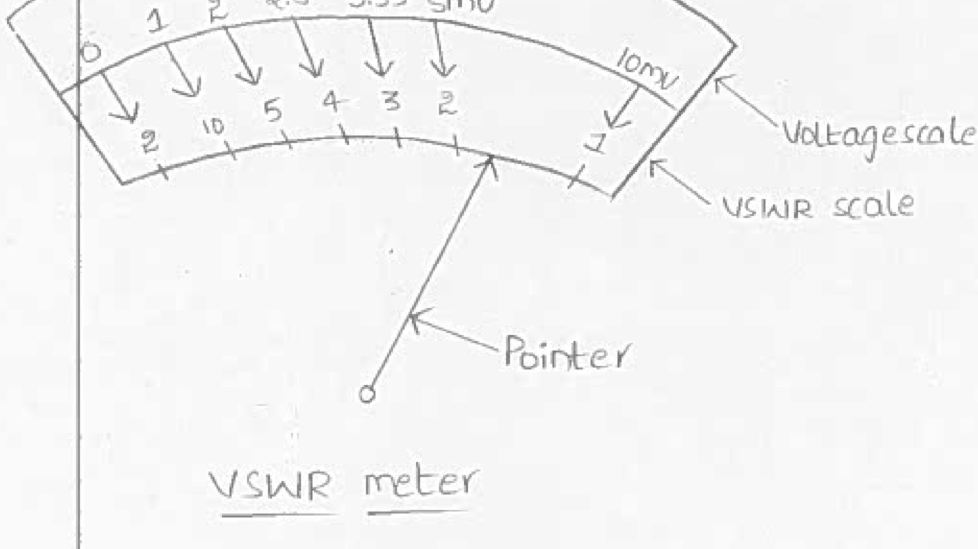
d) Fixed waveguide matched detector mount




a) Tunable probe

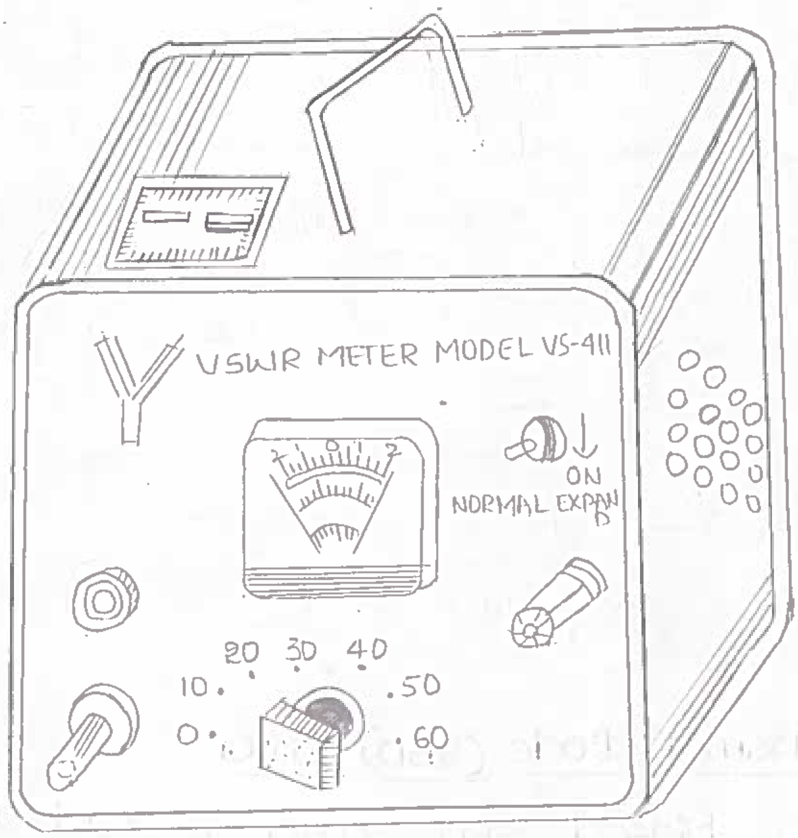


c) Tunable waveguide detector




VSWR meter

The actual VSWR meter look like before. It has a gain control to adjust the reading to a desired value, by fine or coarse adjusting knobs. Normally, the overall gain is about 125dB that can be adjusted in steps of 10. Also there are 3 scales on the VSWR meter- normal SWR, expanded SWR, and dB scales. The normal SWR scale can be used when the VSWR is between 1 and 4.

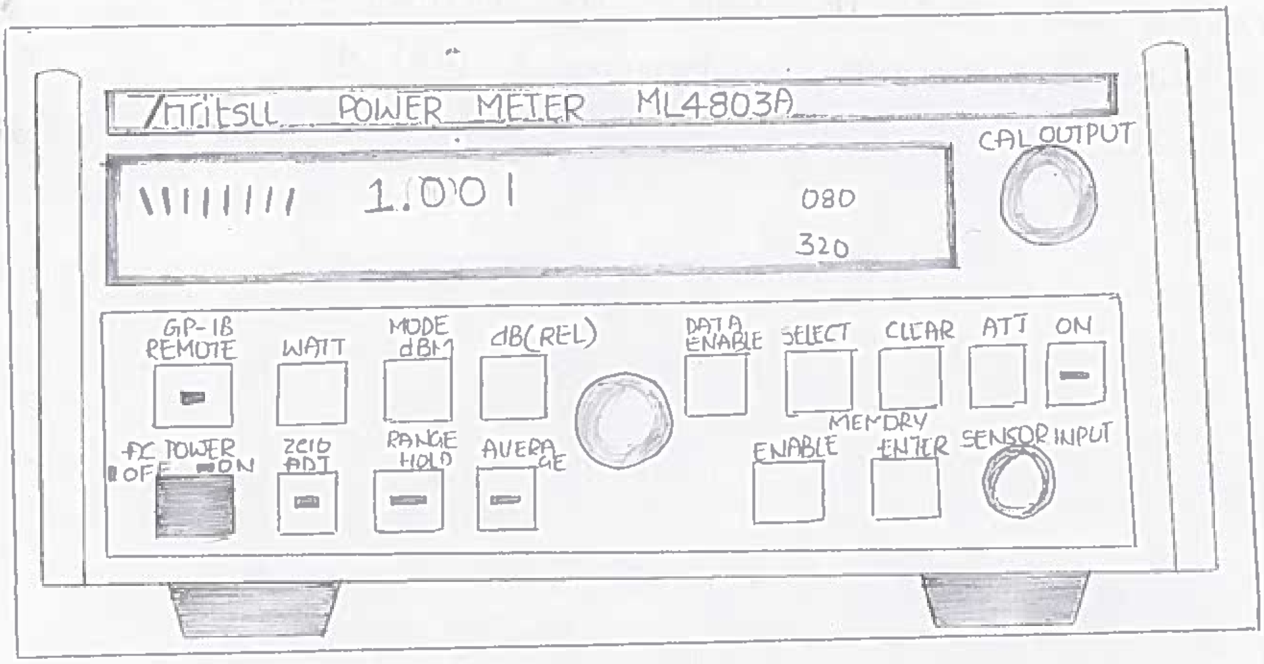



VSWR meter

For VSWR's between 3 and 10 the bottom of normal SWR scale can be used. The expanded SWR scale is graduated from 1 to 1.3 and hence can be used whenever the VSWR is less than 1.3 for an accurate reading. The dB scale is at the bottom along with an expanded dB for measuring VSWR directly in dBs. The input selector switch is provided for different inputs - crystal low current (4.5mA) or high current (8.75mA) balometer bias.

iv) Power Meter

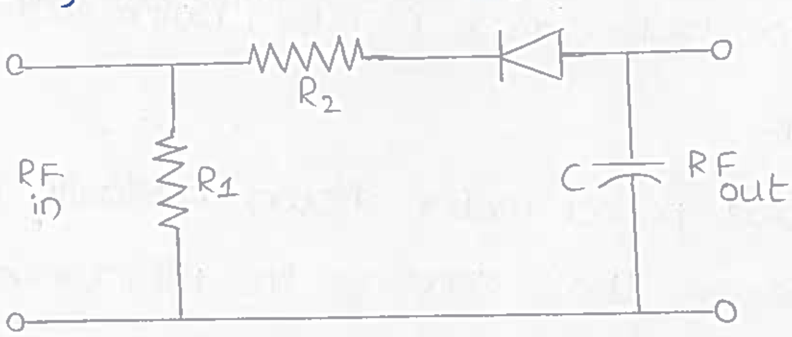
A microwave power meter shown basically consists of a power sensor that converts the microwave power into heat energy. The temperature change so produced provides an output current in the low frequency circuit that indicates power. The sensors used for power measur-ements are the Schottky barrier diode, bolometer and the thermocouple. Here we discuss the Schottky barrier diode and the thermocouple sensor.




a) Power meter

## Schottky barrier diode (SBD) sensor

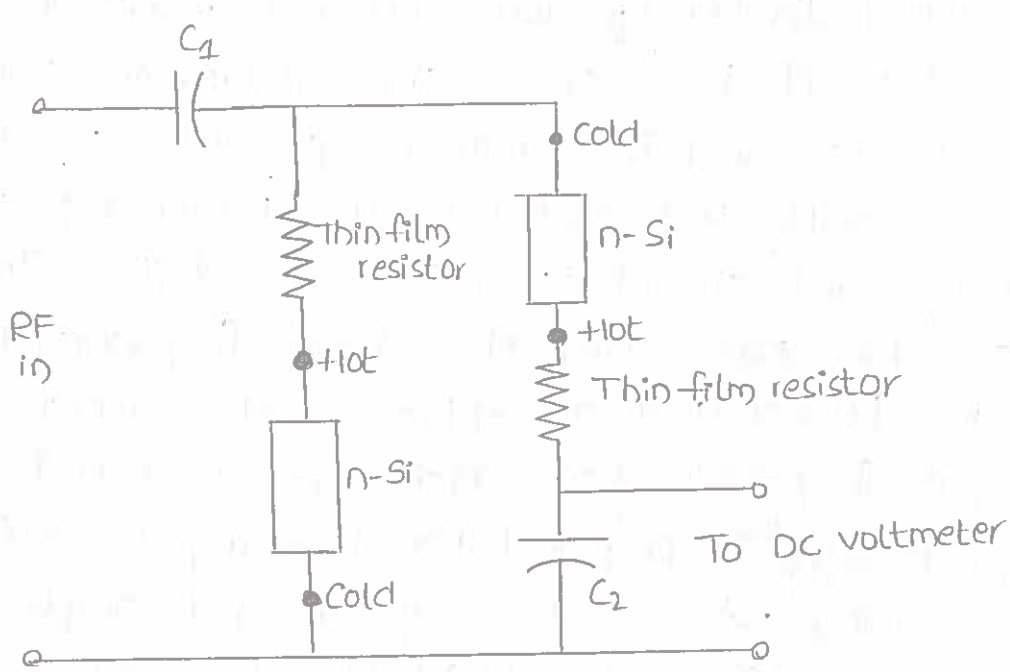
A zero biased SBD sensor is used as a square low detector for measurement of power. The SBD sensor circuit is shown. wherein the circuit is so defined that the input matching is not affected by diode resistance. The output of the circuit is proportional to the input RF power. These SBD detectors can be used to measure power levels as low as 70dBm, meaning that they can be employed to measure only low microwave powers. (less than 10mW)




Schottky barrier diode sensor

## Thermocouple sensor

A thermocouple power sensor can also be used to measure microwave power. We know that when the two ends of a thermocouple are heated up differently by absorption of microwaves they produce an emf across them. This is done in a thin film tantalum-nitride resistive load deposited on a silicon substrate which forms one electrode of the thermocouple. The emf produced will be proportional to the RF microwave input power being measured. Capacitor $C_2$ is an RF bypass capacitor and capacitor $C_1$ is the input coupling capacitor for dc blocking. The figure shows dual thermocouples that are parallely connected. The overall emf generated by these thermocouples get added and appear across $C_2$ that is connected to a dc voltmeter. The dc voltmeter can be calibrated to read the input microwave power directly. Generally, the microwave power is square wave modulated. If the average power of this signal is known, the peak power can be calculated since $P_{peak} = P_{av} \cdot T / \tau$ where $\tau$ is the pulsewidth, T is the period and $P_{av}$ is the average microwave power.




c) Thermocouple sensor

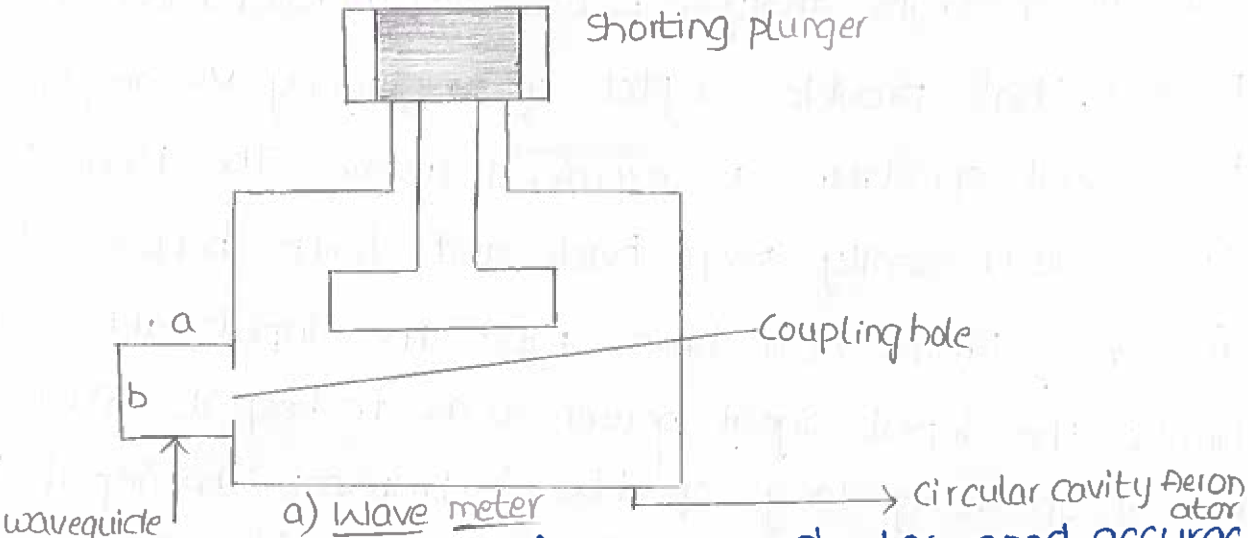
v) Wave Meter

A wave meter is typically constructed of a cylindrical cavity resonator with a variable short circuit termination. The shorting plunger is used to change the resonance frequency of the cavity by changing the cavity length. The wave meter axis is so placed that it is perpendicular to the broad wall of a waveguide.

The dominant mode $TM_{011}$ is normally used in wave meter. The most suitable mode however is $TE_{011}$ because of its higher Q and absence of axial current. The $TM_{010}$ mode is excited in the cavity through the coupling hole by magnetic field coupling. Any possible oscillat-ion due to plunger can be avoided by placing a block of polytron - an absorbing material, at the back of the tuning plunger. The various plunger positions result in different cavity resonant freque-ncies. This tuning can be calibrated in terms of frequency by use of known frequency inputs and observing a dip in the power meter. The power meter can be connected at the output side of the wavegui-de.

Quality factor of 1000-5000 will result in accuracies as small as 1% to 0.005%. The absorption cavity characteristics and its analog equivalent is shown. A resonant cavity wavemeter is the microwave analog to tuned resonant circuit. There are two types - Transmission cavities which pass only the signal frequency to which they are tuned and absorption cavities which attenuate the signal frequency to which they are tuned. The absorption type is prefered for laboraty frequency measure-ment. Cavity wave meters are rugged, simple and highly accurate. Accuracies upto 99.9% can be achieved. The resonant frequency of the cavity wavemeter is determined primarily by the physical dimensions a, b, d and the mode is determined by m, n, p as given by

$$f_0 = \frac{c}{a}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{d}\right)^2}$$

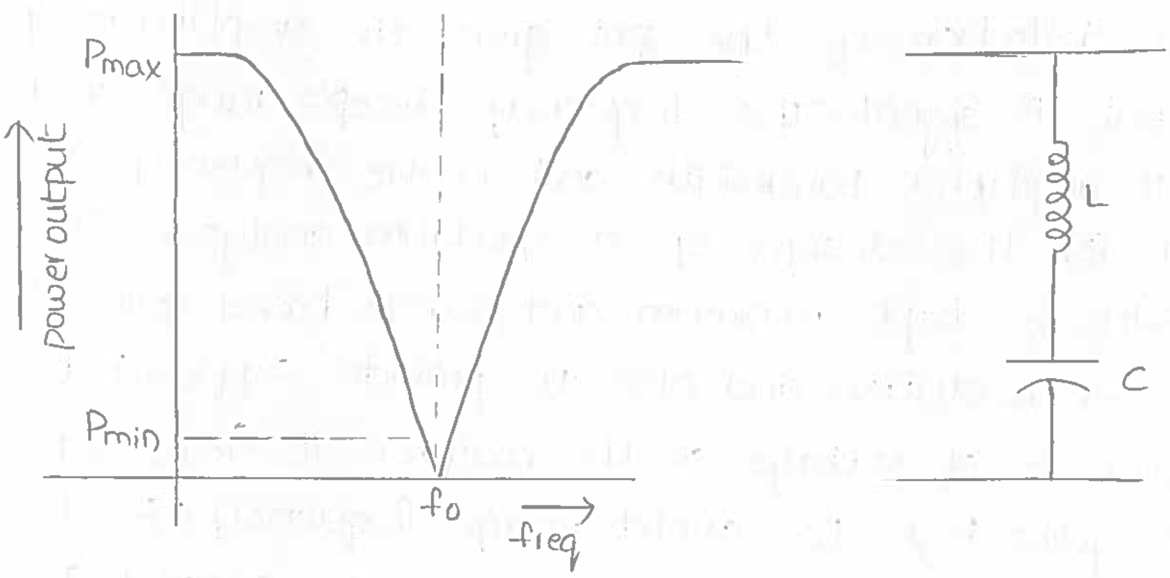



a) Wave meter

They should have high frequency Q's for good accuracy.

A micrometer type frequency meter using absorption type cavity wavemeter is shown.

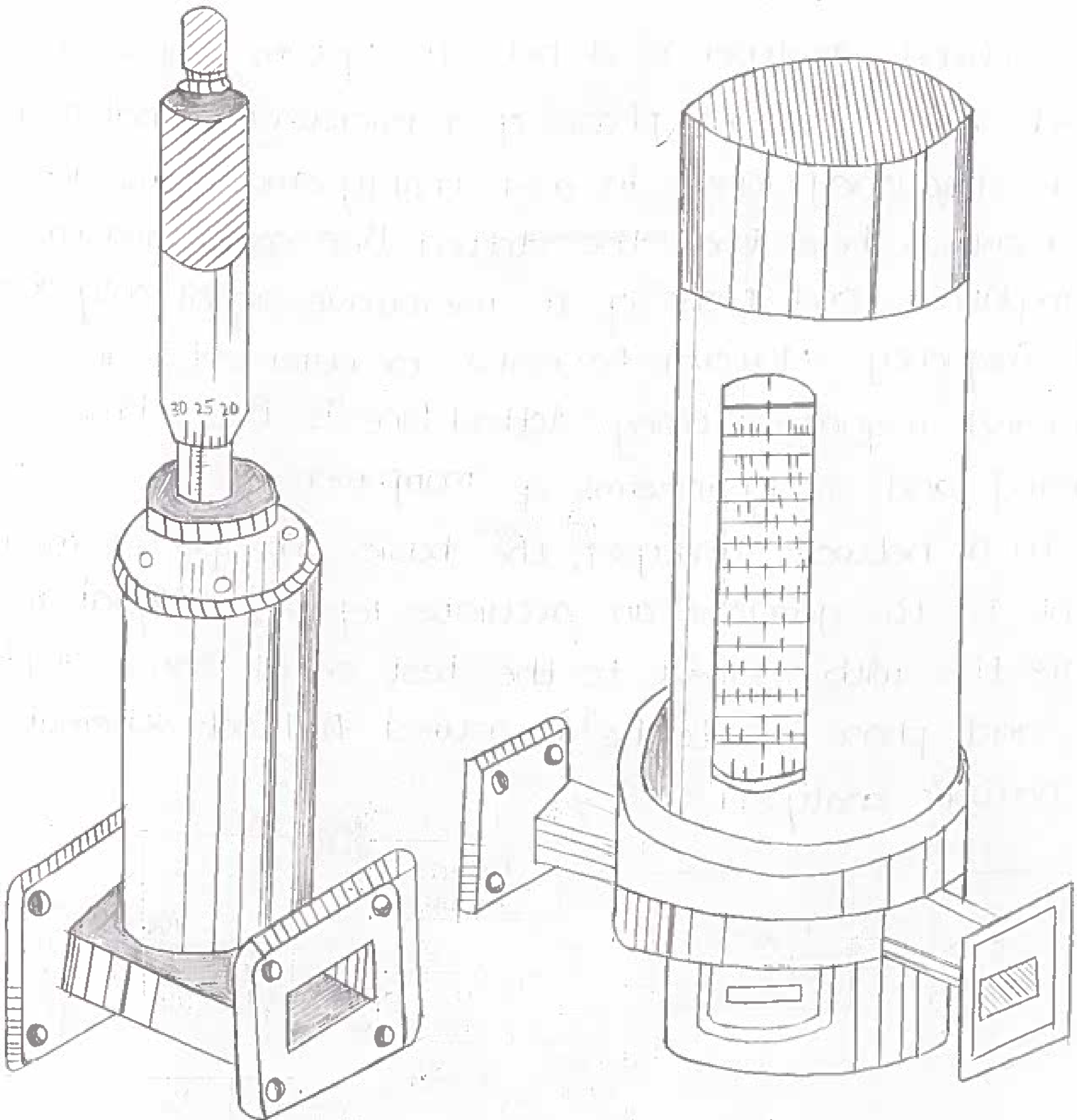
A direct reading frequency meter is shown.




Absorption cavity b) Characteristics, and c) Analog equivalent

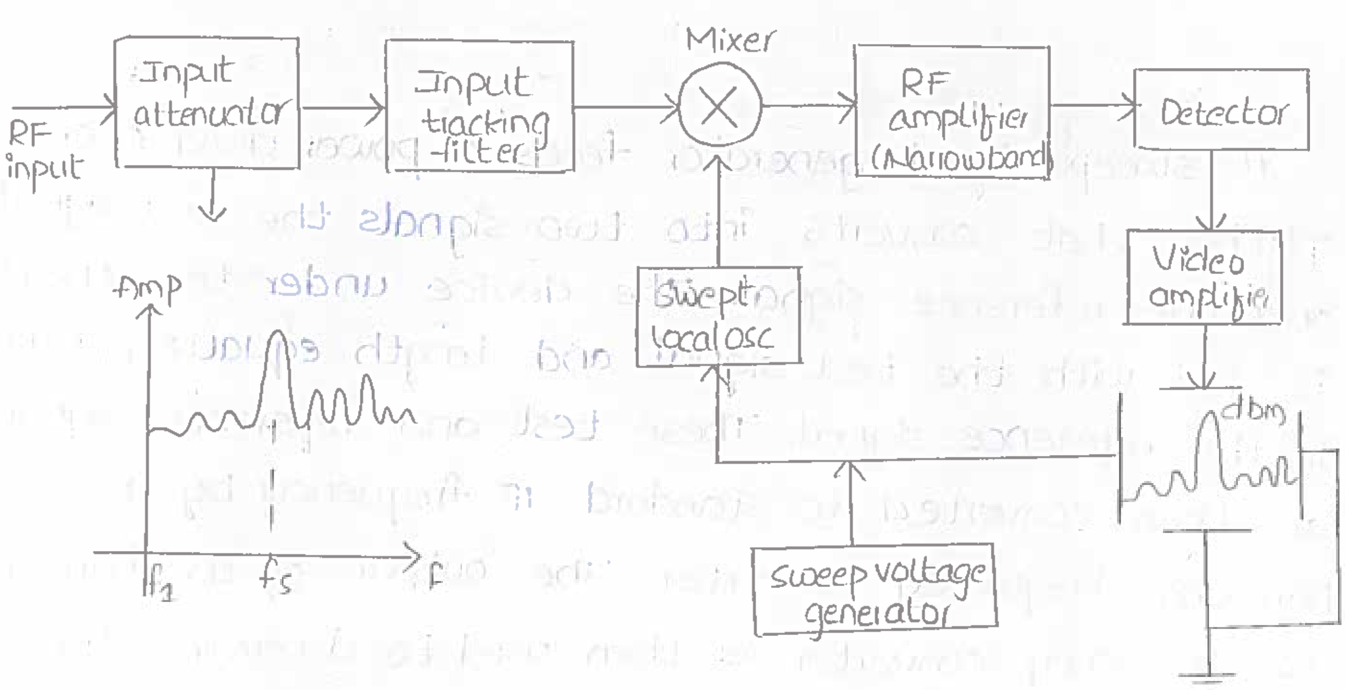
## vi) Spectrum Analyser

A spectrum analyser is a frequency domain instrument which gives a display of the frequency spectrum of the input signal. The spectrum analyser gives a plot of frequency vs signal amplitude. Spectrum analysers are particularly useful at RF and microwave frequencies for analysing the spectrum of the signal source antenna or signal distribution. It also helps as a diagnostic tool to establish compliance for the RFI/EMI requirements. A spectrum analyser is basically a broad band superhet receiver, that provides a plot of frequency vs amplitude i.e., signal spectrum as explained above. The local oscillator is electronically swept back and forth between two frequency limits at a linear rate. The input attenuation limits the input signal power so as to keep it within the normal operating range of the instrument. The input tracking filter provides image frequency rejection. The sawtooth sweep voltage waveform moves the spot on the CRT horizontally. This movement is performed in synchronism with the frequency sweep so that the horizontal position is a function of the frequency of the local oscillator. The vertical deflection of the spot gives the amplitude of the input IF signal. The frequency sweep's range and rate, IF amplifier bandwidth and centre frequency are critical for the design of a spectrum analyser. The bandwidth is kept minimum and sweep based quite low for better resolution and also to provide sufficient time for build up of voltage at the receiver. IF frequency is chosen quite high to avoid image frequency ( $f_{si} = f_s \pm 2f_i$) then the range of frequencies to be covered should be as small as possible. The bandwidth of the IF amplifier determines the bandwidth and hence resolution of the spectrum analyser.


30 25 20

d)

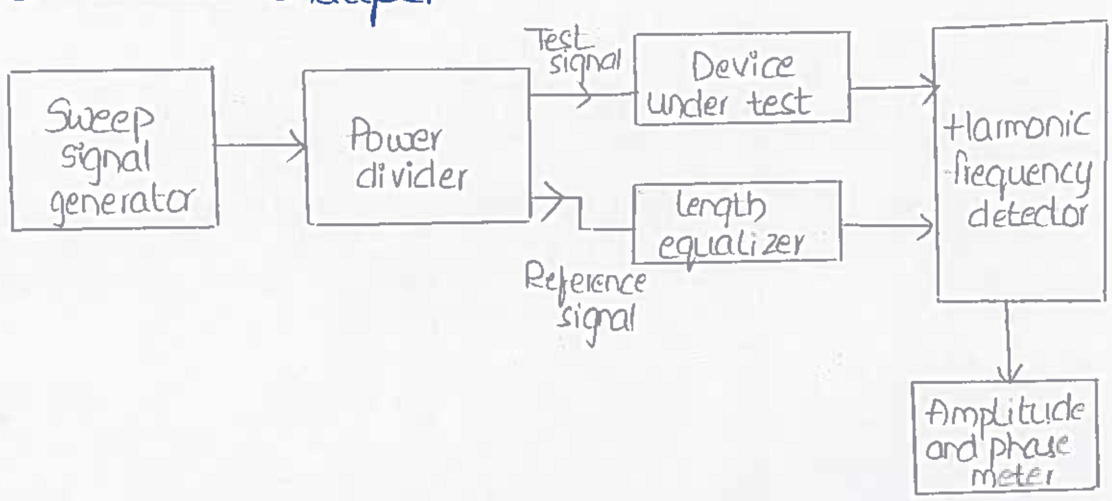
e)


RF input
Input attenuator
Input tracking filter
Mixer
RF amplifier (Narrowband)
Detector
Video amplifier
dbm
Amp
$f_1$
$f_s$
f
Swept local osc
Sweep voltage generator

## VII) Network Analyser

A network analyser is a network system that measures both amplitude and phase of a microwave signal over a wide frequency range in a resonably small time. We may mention here that the slotted line could measure the amplitude and phase of a microwave signal only at a signal frequency. Moreover to make measurements at broad band frequencies using slotted line is both time consuming and costly in terms of manpower.

In a network analyser, the basic principle of measurement is to generate an accurate reference signal and compare this with respect to the test signal whose amplitude and phase are to be measured. A block schematic of a network analyser



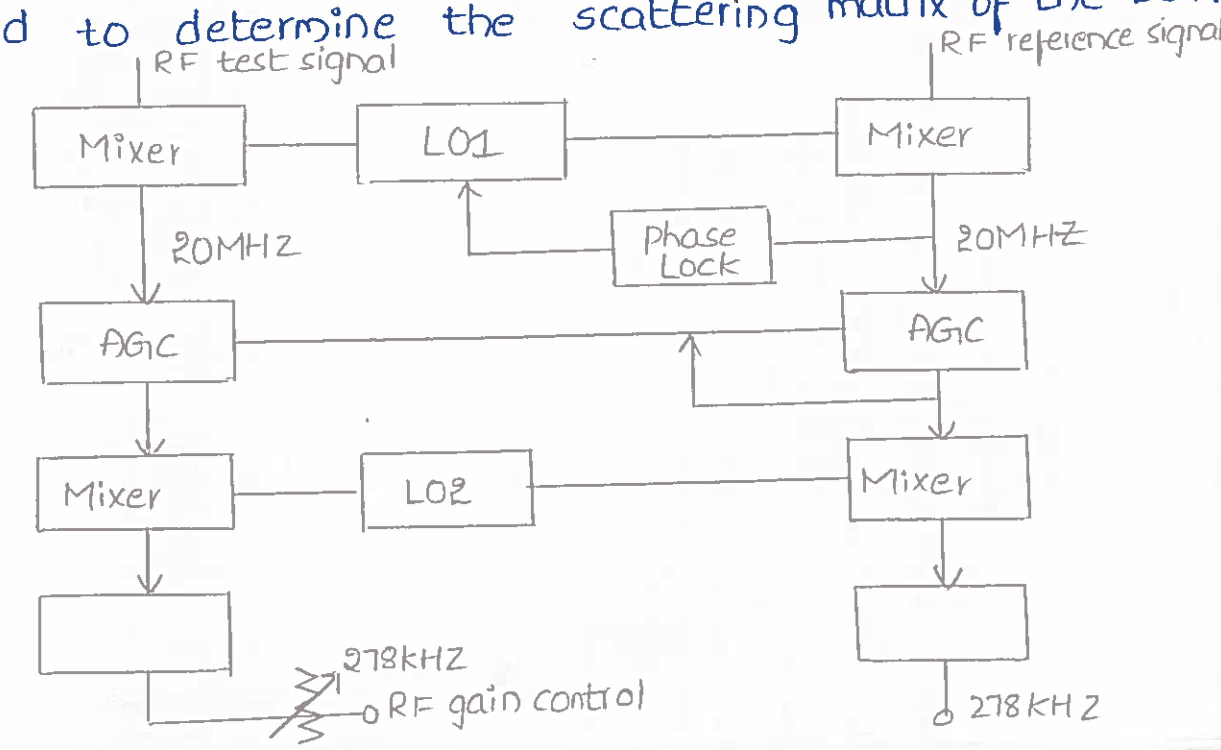

Network analyser

A sweep signal generator feeds a power divider or splitter that converts into two signals the test signal and the reference signal. The device under test (DUT) is fed with the test signal and length equalizer takes in the reference signal. These test and reference signals are then converted to standard IF frequency by a harmonic frequency converter. The output of the harmonic frequency converter is then used to determine the amplitude and phase of the test signal.

The harmonic frequency converter consists of a phase locked loop. The local oscillator tracks the reference signal frequency making error possible swept frequency measurem-ent. The double mixer arrangement first converts the memory RF test signal to a first IF of 10MHZ and then a second IF of 278kHZ.

A network analyser is quite useful for measurement of both passive as well as active microwave component or network parameters. It is used for measurement of both impedance and gain characteristics of microwave devices. It uses a stimulus response method for testing over the frequency range of interest. As these parameters are a function of frequency with complex variables, a swept frequency measurement becomes imminent.

A network analyser can be scalar or vector. A scalar network analyser provides only magnitude characteristics of microwave devices as a function of frequency. A vector network analyser can measure complex reflection or transm-ission characteristics of microwave devices. The principle of measurement in both these methods is the same in that they compare the incident or input power with the transmitter or reflected waves depending upon the parame-ter to be measured. The ratio of the relevant signals is used to determine the scattering matrix of the DUT.



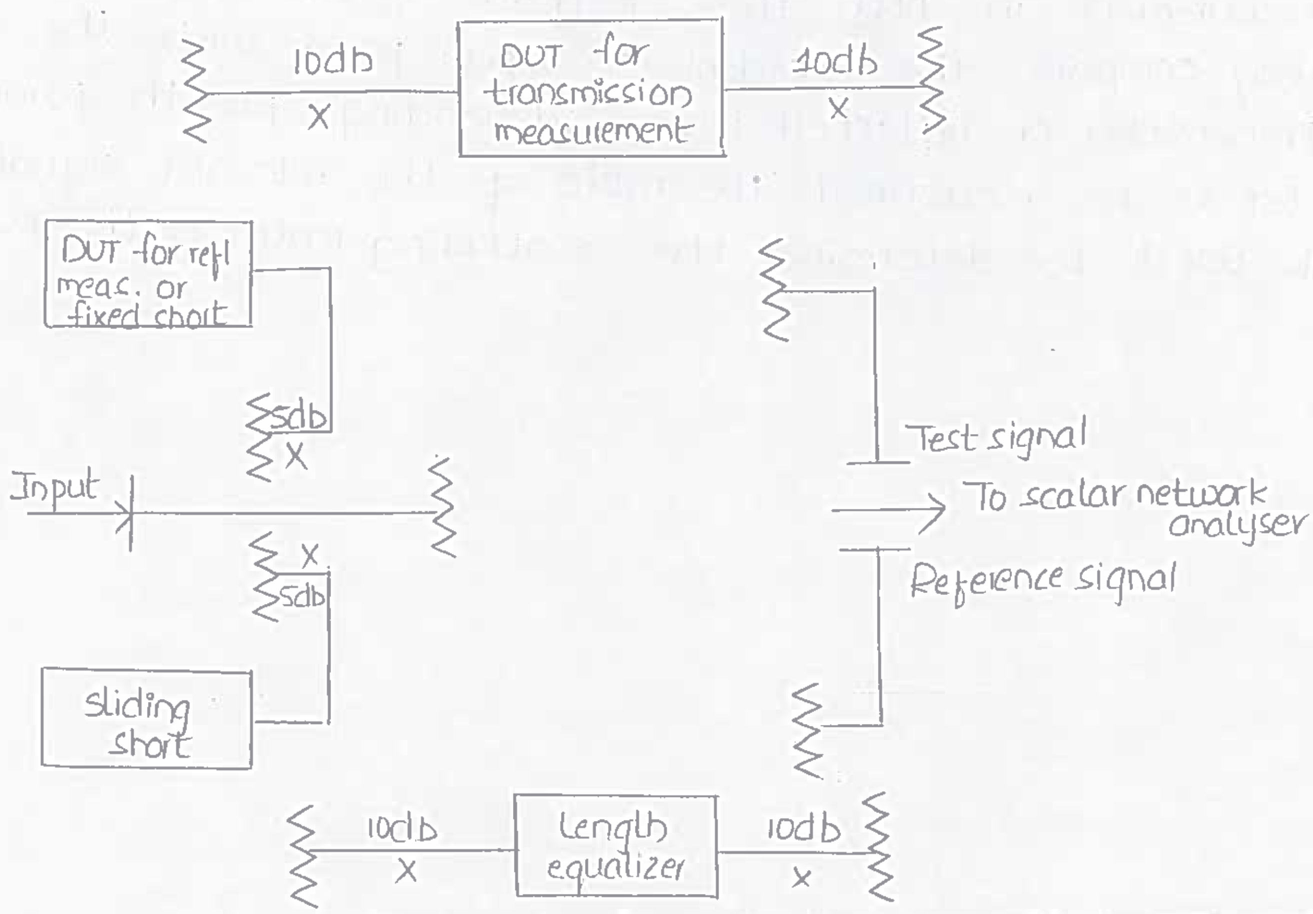

Harmonic frequency converter

As explained earlier, we have a signal source, power splitter, receiver/detector and a processor/display components. The DUT transmits or reflects the stimulating or incident signal from the signal source which are used in the measurement of magnitude and phase of individual components of the test signal. The receiver and display could be a harmonic generator and the amplitude phase meter which have already been described. However in place of power splitter, we might have as well used a directional coupler or a high impedance probe.

VIII) Test Set-Up for Reflection - Transmission Measurement

A reflection - transmission unit is normally employed for measurement of reflection and transmission. A reflection - transmission unit.

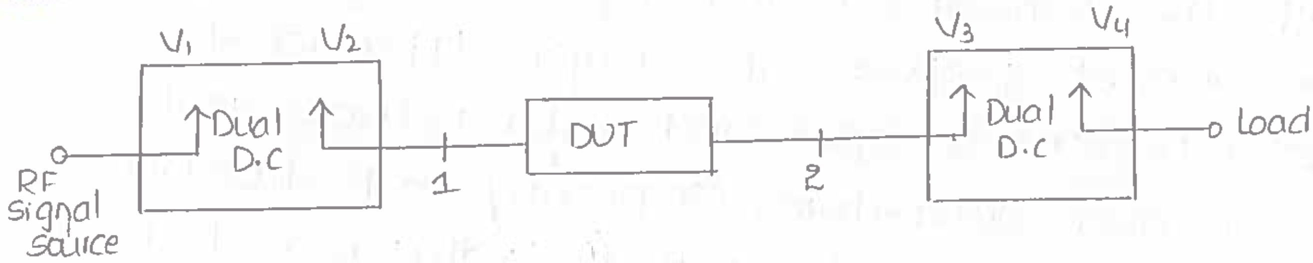
For transmission measurement, the reference line length is balanced and for reflection measurements the DUT is compared to the sliding short. A good balance between the channels can be maintained by use of accurately matched directional couplers in the bridge.



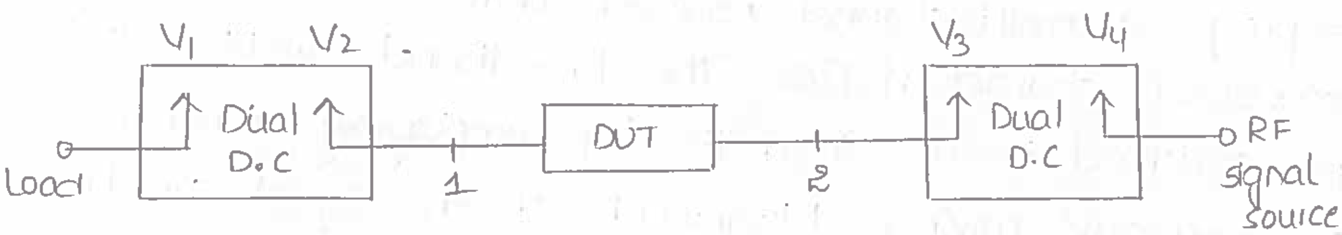

Test setup for reflection-transmitter measurement

## IX) Test Set-up for S-parameter Measurement

It is possible to measure S-parameters using network analyser for a two port network. Such a scheme is shown. The S-parameters of a two port network (DUT) can be obtained by measuring the amplitude and phase from the ports through the dual directional coupler as per diagram by interchanging the RF is signal source and the load positions.




a)




b)

From fig a.

$$S_{11} = \frac{V_2}{V_1}(\phi_2 - \phi_1)$$

$$S_{21} = \frac{V_3}{V_1}(\phi_3 - \phi_1)$$

For measuring $S_{22}$ and $S_{12}$, the RF signal source and load are interchanged as shown in b.

$$S_{22} = \frac{V_3}{V_4}(\phi_3 - \phi_4)$$

$$S_{12} = \frac{V_2}{V_4}(\phi_2 - \phi_4)$$

Thus with a scalar network analyser, it is possible to measure the transmitted and reflected signal magnitudes. Also, it can be employed to characterisize many of the microwave devices such as antennas, amplifiers, RF bridges, attenuators, mixers, couplers, receivers, up/down converters, power-dividers. etc.

A vector network analyser on the other hand can measure complex-transmission-reflection characteristics of several microwave devices as a function of frequen -cy, i.e., it measures both magnitude and phase information. It also compares the incident RF signal with the transmitted and reflected signals for measurement purpose. The major difference between vector network analyser and scalar network analyser is the receiver architecture, complexity and detection technique. In a scalar network analyser, a diode det -ector is employed whereas the vector network analyser employs a multichannel receiver that is linear in its conversion characteristics. The broadband swept signal is converted to a fixed IF by employing fundamental or harmonic mixing. Magnitude of the signal can be measured in each of the receiver channels while phase relationship between any two receiver channels could also be measured. Thus complex impedances, phase delay characteristics, electric delay, group delay and distance to fault in transmission structure measuremen ts can be easily performed using vector network analyser.

## Measurement of Attenuation

Microwave components and devices almost always provide some degree of attenuation. Attenuation is the ratio of input power to the output power and is normally expressed in decibels

i.e., $\text{Attenuation (in dBs)} = 10\log \frac{P_{in}}{P_{out}} \rightarrow (1)$

where $P_{in}$ = input power

$P_{out}$ = output power

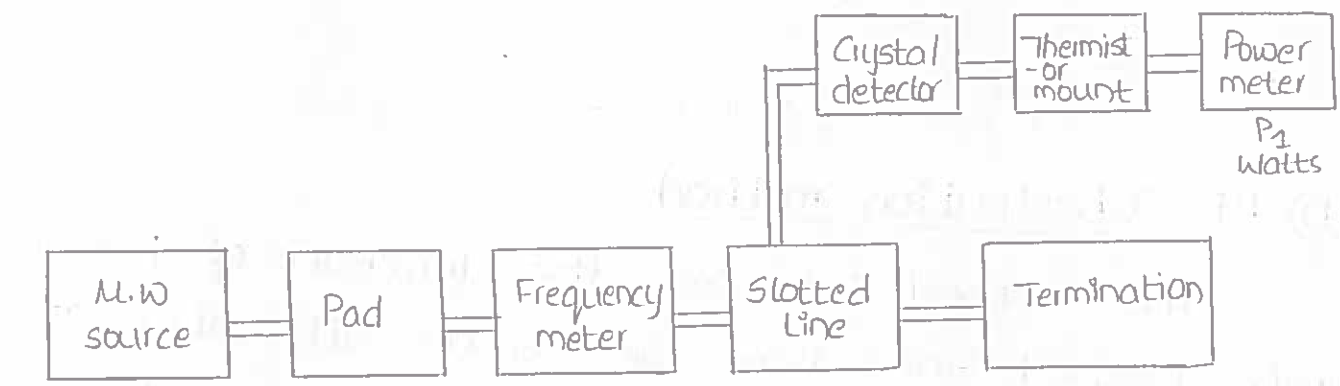
The amount of attenuation can be measured by two methods.

a) Power Ratio method

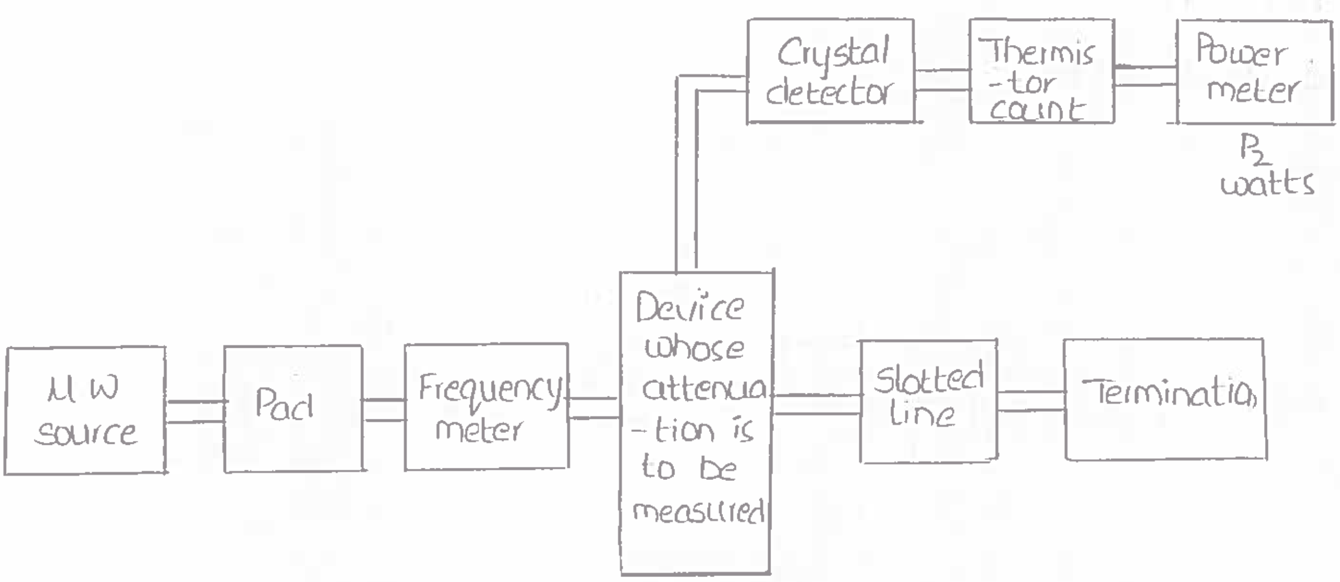
b) RF substitution method.

a) Power Ratio Method

This method involves measuring the input power and output power with and without the device whose attenuation is to be measured as shown in setup 1 and setup 2. The powers are measured in each setup as $P_1$ and $P_2$. The ratio of power $P_1/P_2$ expressed in decibels gives the attenuation as in eq ①.

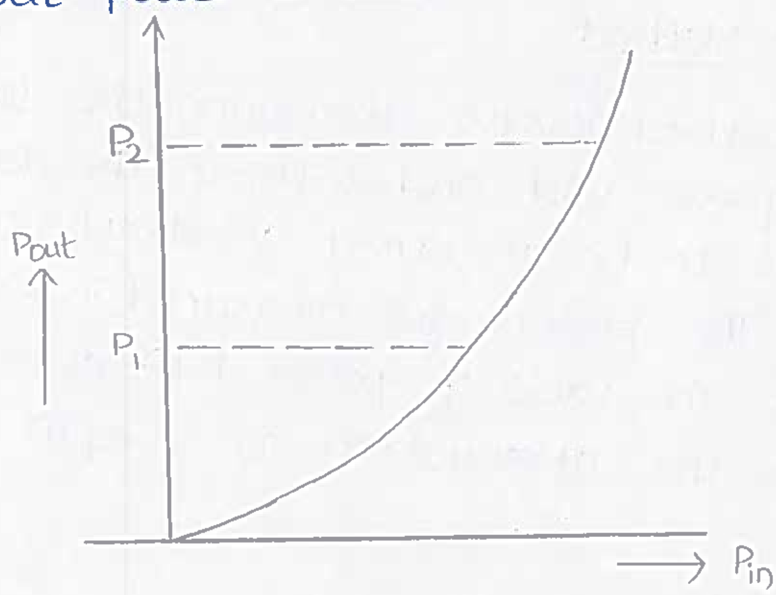



Setup 1, power ratio method




Setup 2, power ratio method

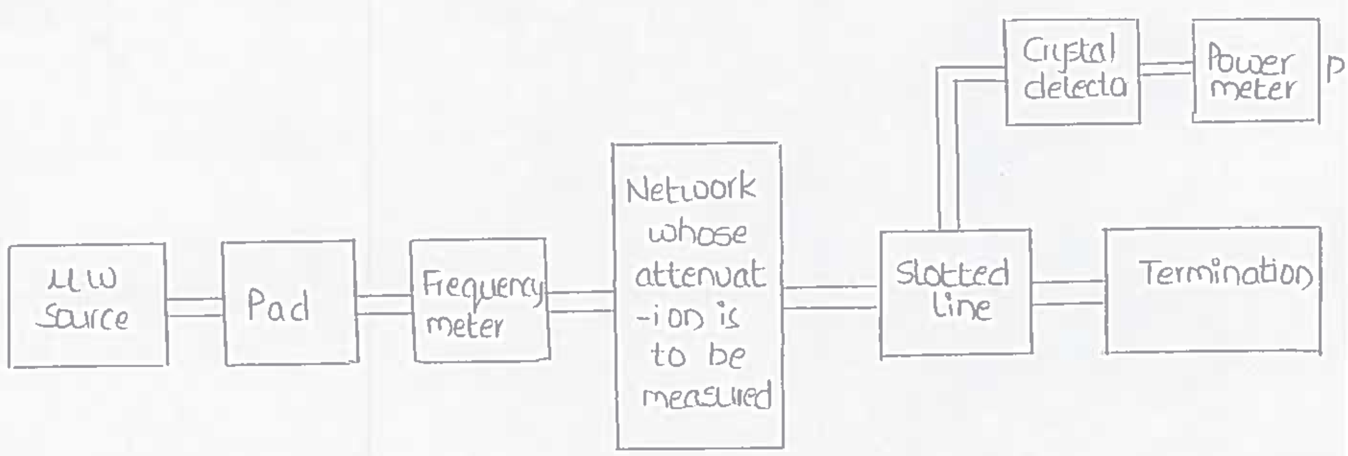
The drawback of this method is that the attenuation measured corresponds to two power positions on the power meter with a square law crystal detector characteristics as shown in below fig. Due to non-linear characteristics the two powers measured and the attenuation calculated will not be accurate particularly if the attenuation of the network is large and if the input power is low.




Square law characteristics of crystal diode

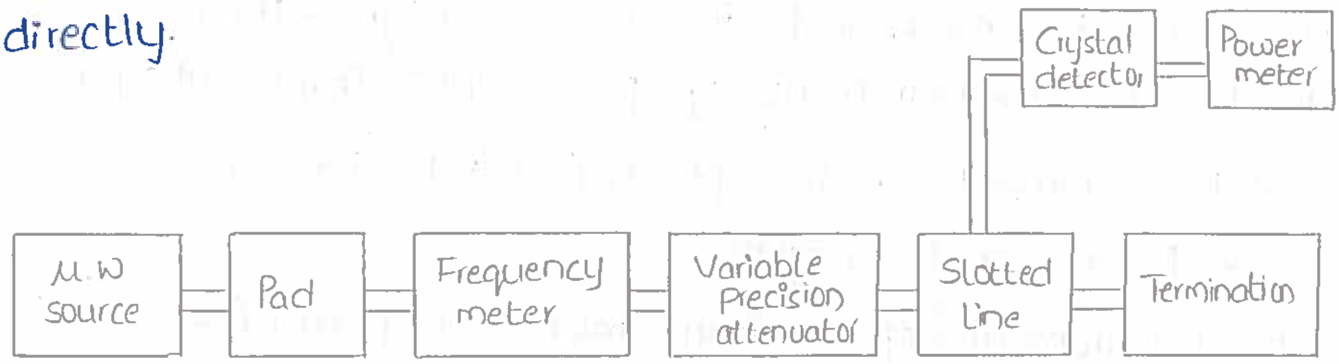
## b) RF Substitution method

This method overcomes the drawback of power ratio method since here we measure attenuation at a single power position. This method consists of measuring the output power say 'P' by including the network whose attenuation is to be measured in setup 1 as shown.




Setup 1, RF substitution method

In setup 2 this network is replaced by a precision calibrated attenuator which can be adjusted to obtain the same power 'p' as measured in setup 1. Under this condition the attenuation read on the precision attenuator would give attenuation of the network directly.



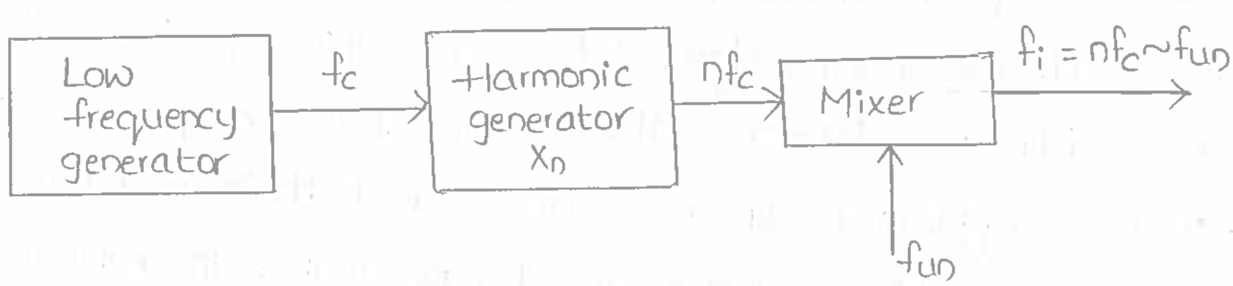

Setup 2, RF substitution method

## Frequency, Standing Wave Measurements

Microwave frequency can be measured by either electronic or mechanical techniques.

### Electronic Techniques:

These techniques generally are more accurate but expensive. Frequency counters or high frequency hetero dyne systems can be used. Here the unknown freque-ncy is compared with harmonics of a known lower frequency is compared with harmonics of a known lower frequency by use of a low frequency generator, a harmonic generator and a mixer.




Measurement of Frequency-electronic method

## Measurement of power

The microwave power inside a waveguide is invariant with position of measurement and the power measured is the average power. The technique used to measure power depends on whether the power to be measured is low or high. Thus we divide the measurement of power into three categories.

a) Measurement of low power (0.01mW - 10mW)
- Bolometer technique.

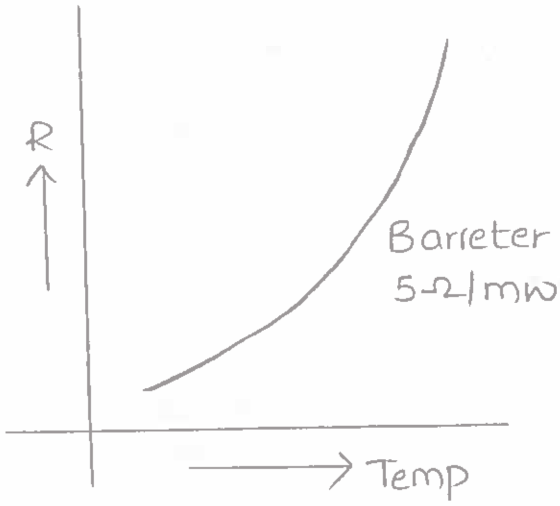
b) Measurement of medium microwave power (10mW - 1W)
- Calorimeter Technique

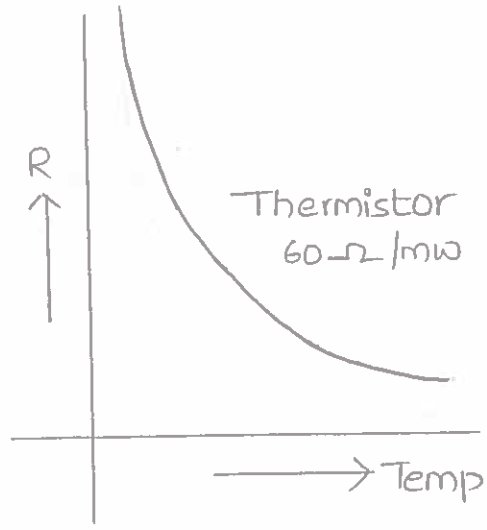
c) Measurement of high microwave power (>10W)
- Calorimeter Watt meter.

### a) Measurement of low microwave power

Devices such as bolometers and thermocouples whose resistance changes with the applied power are capable of measuring low microwave powers. Bolometers are most widely used among these.

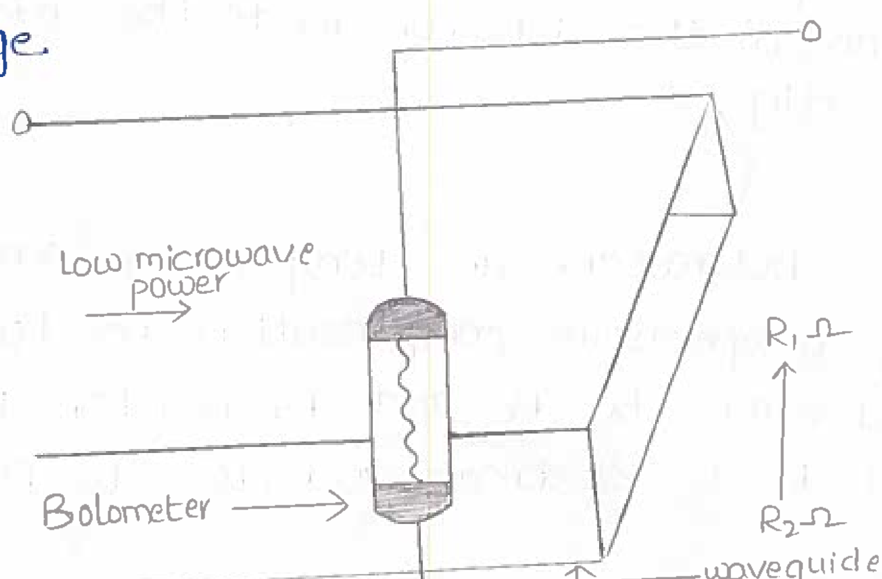
Bolometer is a simple temperature sensitive device whose resistance varies with temperature. These are of two types viz Barretters and Thermistors. Barretters have positive temperature coefficient and their resistance increases with an increase in temperature. It basically consists of a short length of fine Platinum wire mounted in a catridge, like an ordinary fuse. It is very delicate device. Thermistors have negative temperature coefficient of resistance and their resistance decreases with increase in temperature. Thermistors are basically semiconductor materials.

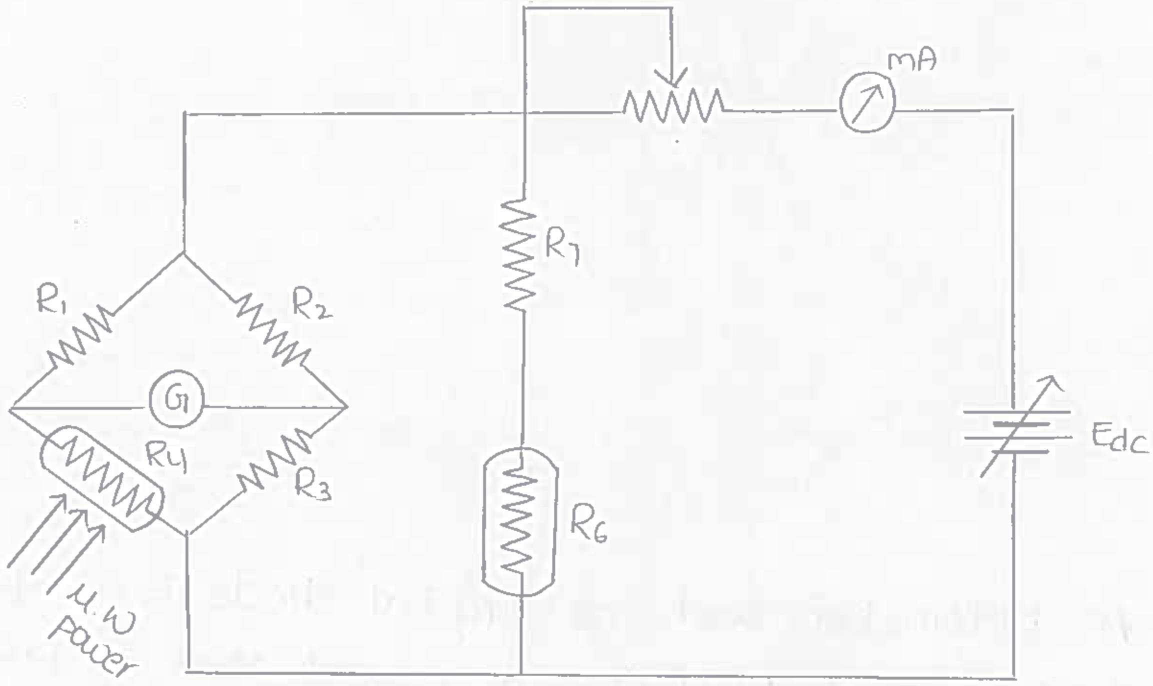



A bolometer such as crystal diode is a square law device and it produces a current that is proportional to the applied power i.e., square of the applied voltage, rather than the applied voltage. Bolometer is mounted inside the waveguide, where the bolometer itself is used as a load, with the operation resistance as $R_1\Omega$. Now the low microwave power which is to be measured is applied. Some power is absorbed in the bolometer load and dissipated as heat and the resistance changes to $R_2$. This change in resistance $(R_1 \sim R_2)$ is proportional to the microwave power which can be measured using a bridge. Inaccuracy is introduced due to bolometer non-linear characteristics.

In the balanced bolometer bridge technique, the bolometer itself is made to be one of the arms of the bridge.

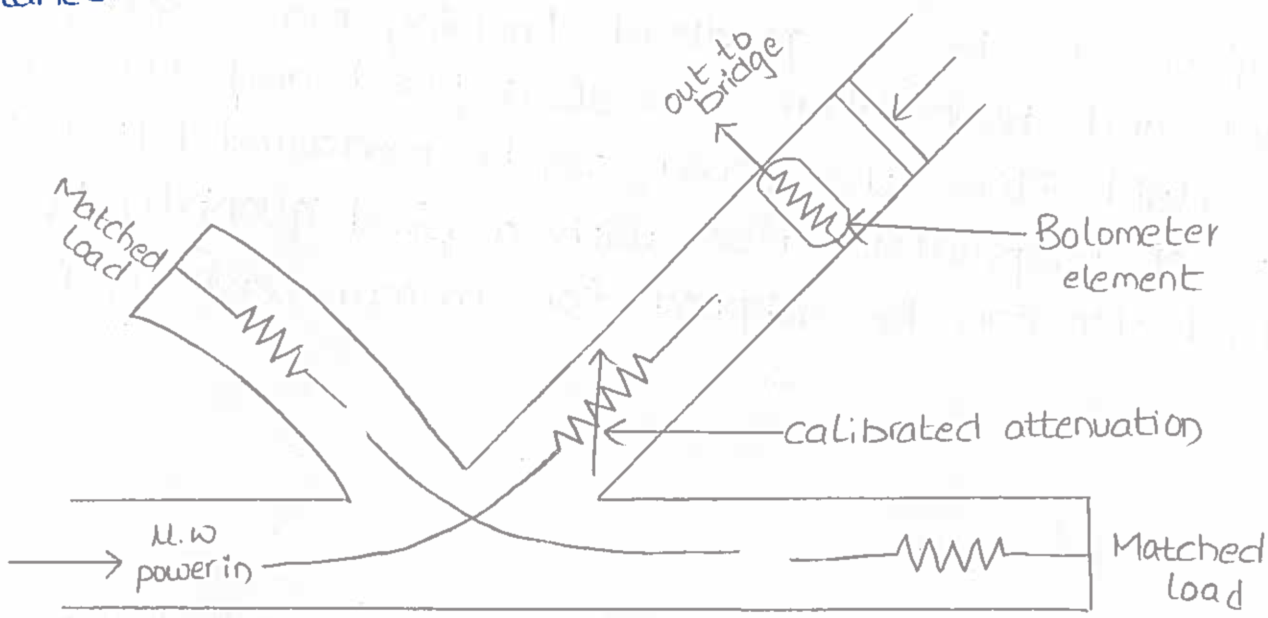





Initially, the bridge is balanced by adjusting $R_5$, which varies the dc power applied to the bridge and the bolometer element is brought to a predetermined operating resistance before microwave power is applied. Let the voltage of the battery be $E_1$ at balance. The microwave power is now applied and this power gets dissipated in the bolometer. The bolometer heats up and it changes its resistance. Therefore the bridge becomes unbalanced. The applied dc power is changed to $E_2$ to get back the balance and this change in dc battery voltage $(E_1 \sim E_2)$ will be proportional to the microwave power. Alternately the detector 'G' can be directly calibrated in terms of microwave power so that when the bridge is unbalanced, the detector reads the microwave power directly.

<u>Errors:</u>

Since bolometers are temperature sensitive some form of temperature compensation has to be used to avoid errors. By $R_6$ and $R_7$ resistors this can be achieved. $R_6$ is identical and close to $R_3$ i.e., both are bolometer elements as shown and subjected to the same ambient temperature. If temperature changes and reduces the resistance $R_3$ this will not be interpreted as a microwave power change because the resistance $R_6$ will be equally reduced. Thus more current will flow through it and hence lesser amount will flow through the bridge i.e., the current through $R_3$ will be lowered and so will its temperature thus restoring the balance.



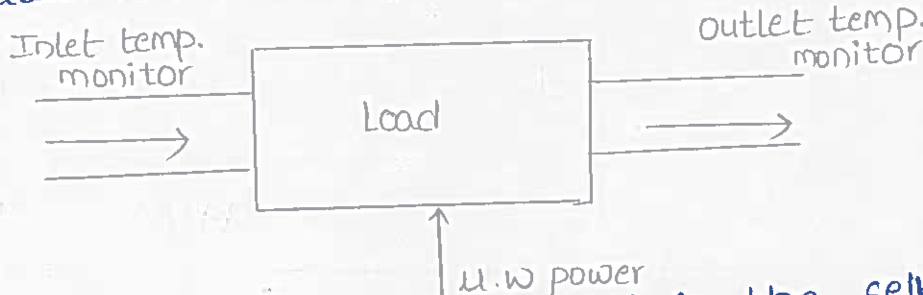

## Limitations:

Barretters and Thermistors both are limited in their power handling ability to about 10mW, so that powers greater than 10mW cannot be measured with them directly. However power measurement range can be increased by using a directional coupler.

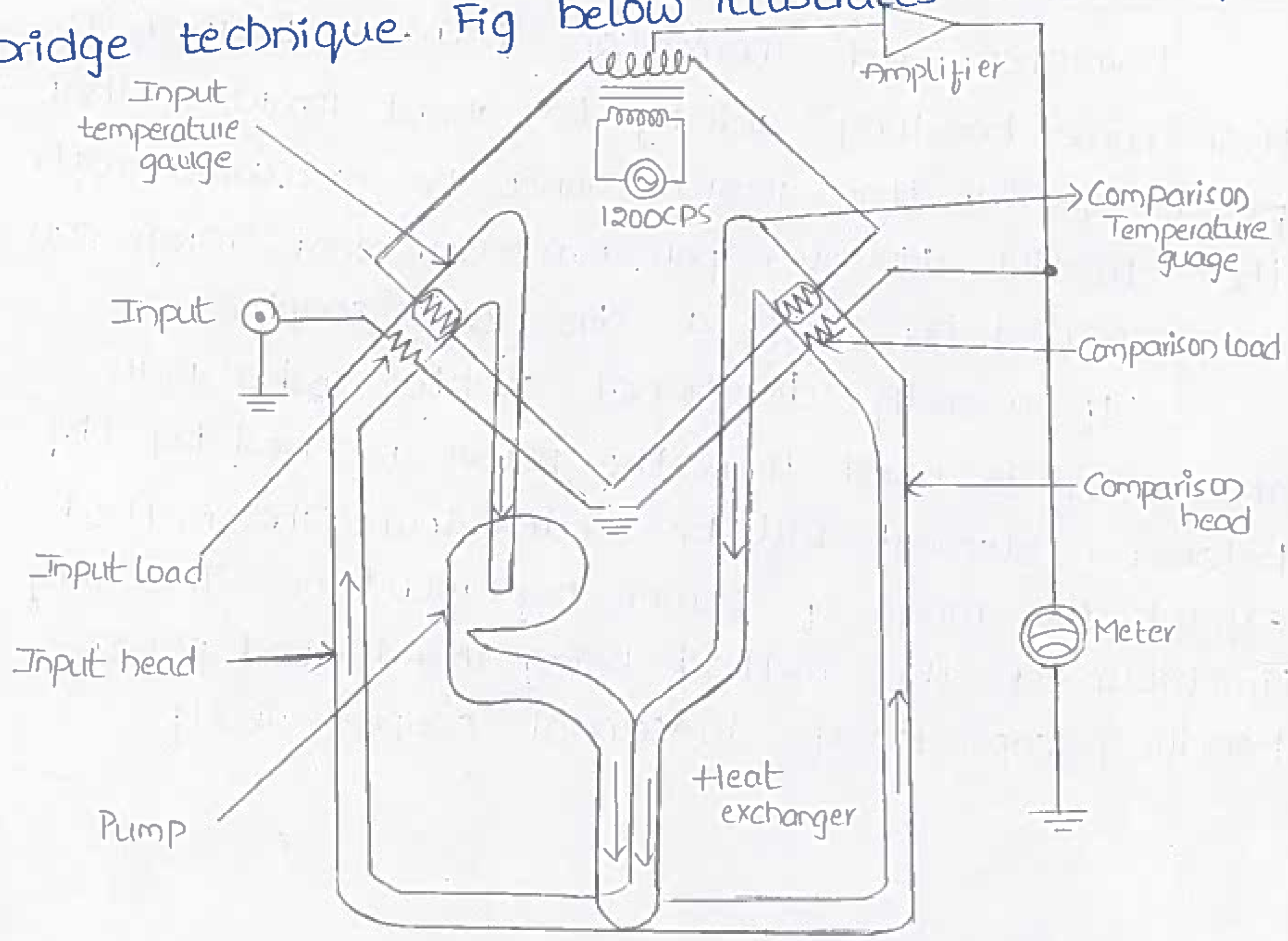
If a 20dB directional coupler and a 10dB attenuator is used then the power received by the bolometer element will be 30dB down. This method extends the range of power by 1000 times. The only limitation of this method being the limited power handling capacity of directional coupler itself

b) Measurement of medium power

Medium power as already stated is in the range of 10mW to 10W. Such powers can be measured by calorimetric techniques. The principle is very simple wherein the temperature rise of a special load monitored in ~~the~~ which is proportional to the power responsible for the rise as shown. The special load must necessarily have high specific heat. water happens to be a goodload. knowing mass, specific heat and temperature rise at a fixed and known rate of fluid flow, the power can be measured. Alternately rate of temperature rise with a fixed quantity of fluid also can be adopted for measurement of power.




The normally used method is the self balancing bridge technique. Fig below illustrates this technique.



It consists of identical temperature sensitive resistors or guages in two arms, an indicating meter and two load resistors. The input load resistor senses the unknown input microwave power and the comparison head is associated with the comparison power. The input load power and input temperature guage are placed close to each other so that the head generated in the input load resistor raises the temperature of the guage. This results in unbalanci-ng the bridge. The signal due to the imbalance is amplified and then applied to the comparison load resistor which is placed closer to the comparison guage. Hence, the heat generated in comparison load resistor is transferred to its guage and the bridge is rebalanced. The meter measures the amount of power that is supplied to the comparison load in order to rebalance the bridge. It can be calibrated directly in terms of input microwave power. It is necessary that the characteristics of the two guages be matched and also the heat transfer characteristics from each load be same for equal power dissipation in the two loads.

For quick balancing of the bridge and for efficient heat transfer from loads to the guages, the components are immersed in an oil stream. Since the flow rates are the same through the two heads, the accura-cy of power measurement is within ±5%. To maintain constant temperature, the streams are passed through a parallel flow heat exchanger just before they enter the heads. The error signal is amplified by an ampli-fier and the 1200HZ source and meter seperated by means of a transformer which form the orther 4 arms of the bridge

c) High Power:

Any power between 10W to 50KW is considered high power. These are normally measured by calorimetric watt meters These meters can be either dry type or flow type.

A dry type calorimeter normally consists of a co-axial cable which is filled by a dielectric with a high hysterisic loss. The flow type uses circulating water, oil or any liquid which is a good absorber of microwaves. The fluid after flowing through the load experiences a temperature rise due to microwave energy. The difference between the temperature ($T_1$) of known quantity of liquid before entering the load and the temperature ($T_2$) after it emerges is a measure of the power which has been absorbed. knowing the rate of the fluid flow the exact value of power can be calculated by using the equation.

$$P = \frac{Rk\rho(T_2 - T_1)}{4.18} \longrightarrow ①$$

where, P = measured power in watts.

R = rate of flow in ($cm^3/s$)

k = specific heat in cal/g.

$\rho$ = specific gravity in $g/cm^3$

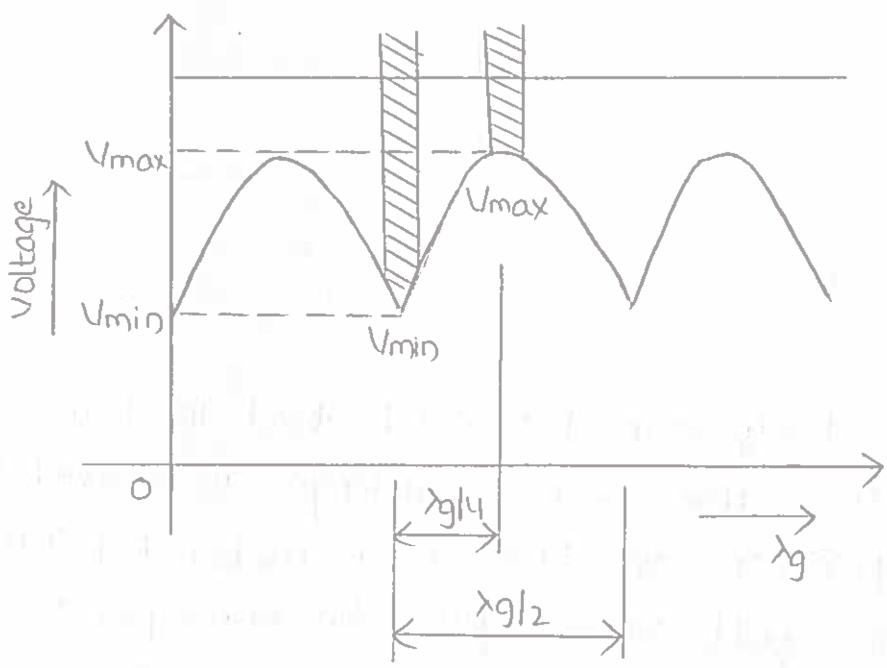
and ($T_2 - T_1$) is the temperature difference in °c.

It may be noted that in calorimeter measurements heat losses do occur due to conduction and radiations, resulting in erroneous measurement of power. Also errors in flow determination, calibration and thermal inertia etc cannot be neglected for accurate measurement.

## Measurement of Low and High VSWR

Any mismatched Load leads to reflected waves resulting in standing waves along the length of the line. The ratio of maximum to minimum voltage gives the VSWR

i.e., $S = \frac{V_{max}}{V_{min}} = \frac{1+\rho}{1-\rho}$




Where, $\rho$ = reflecton coefficient = $\frac{P_{reflected}}{P_{incident}}$

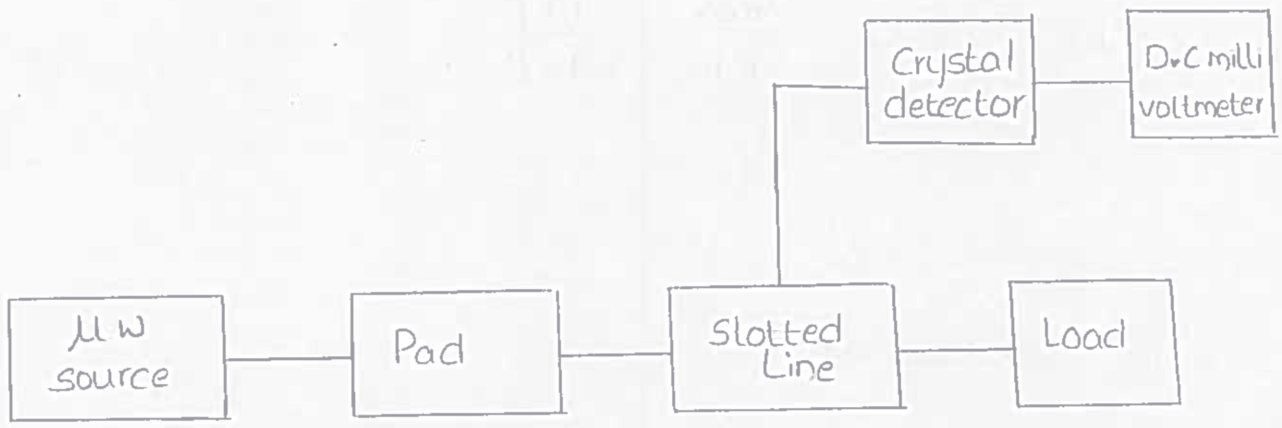
S varies from 1 to ∞

as $\rho$ varies from 0 to ∞

Hence minimum value of S is unity.

a) Measurement of Low VSWR's (S>10)

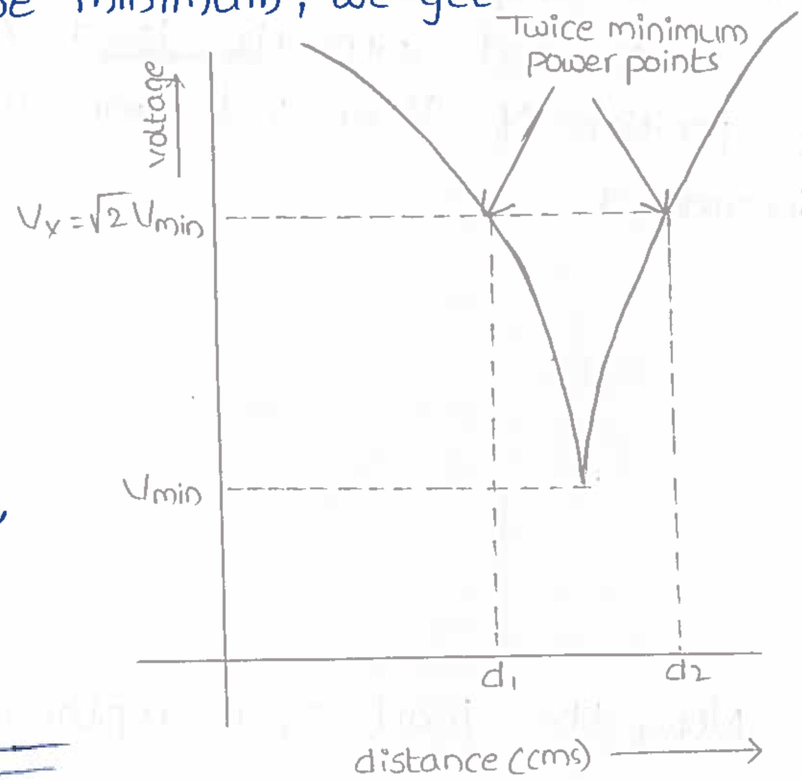
Values of VSWR not exceeding 10 are very easily measured with the setup and can be read off directly on the VSWR meter calibrated. The measurement basically consists of simply adjusting the attenuator to give an adequate reading on the meter, which is a D.C millivolt meter. The probe on the slotted waveguide is moved to get maximum reading on the meter. The attenuation is now adjusted to get full scale reading. This full scale reading is noted down. Next the probe on the slotted line is adjusted to get minimum reading on the meter. The ratio of first reading to the second gives the VSWR.




The meter itself can be calibrated in terms of VSWR. In this case, the probe carriage is moved to give maximum deflection on the VSWR meter by adjusting the pad. This full scale deflection corresponds to a VSWR of 1. As an example, a FSD of 10mV corresponds to a VSWR of 1. The travelling probe is adjusted to get minimum reading on the meter. If this corresponds to 5mV, then $VSWR = \frac{10mV}{5mV} = 2$. If it is 3.3mV, VSWR = 3, If it is 2.5mV, VSWR = 4. If it is 1mV, VSWR = 10 etc. i.e., Such a calibrated VSWR meter gives an expanded scale upto an VSWR of 2 but for VSWR>10, the meter will be congested and the measurement will not be accurate for VSWR's >10. Hence this method is not useful for VSWR's >10.

b) Measurement of High VSWR (S>10)

For VSWR's >10, we use what has come to be known as the double minimum method. In this method, the probe is inserted to a depth where the minimum can be read without difficulty. The probe is then moved to a point where the power is twice the minimum. Let this position be denoted by $d_1$. The probe is then moved to twice the power point on the other side of the minimum, we get




$$2 P_{min} \propto V_x^2$$

$$\therefore \quad \frac{1}{2} = \frac{V_{min}^2}{V_x^2}$$

or $$V_x^2 = 2 (V_{min})^2$$

or $$V_x = \sqrt{2}\, V_{min}$$

Further for $TE_{10}$ mode,

$$\lambda_c = 2a$$

$$\lambda_0 = c/f$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - (\lambda_0/\lambda_c)^2}}$$

Then, VSWR can be calculated using the empirical relation

$$VSWR = \frac{\lambda_g}{\pi (d_2 - d_1)}$$

## Cavity Q, Impedance Measurements

Impedance at the microwave frequencies can be measured using any of the following 3 methods.

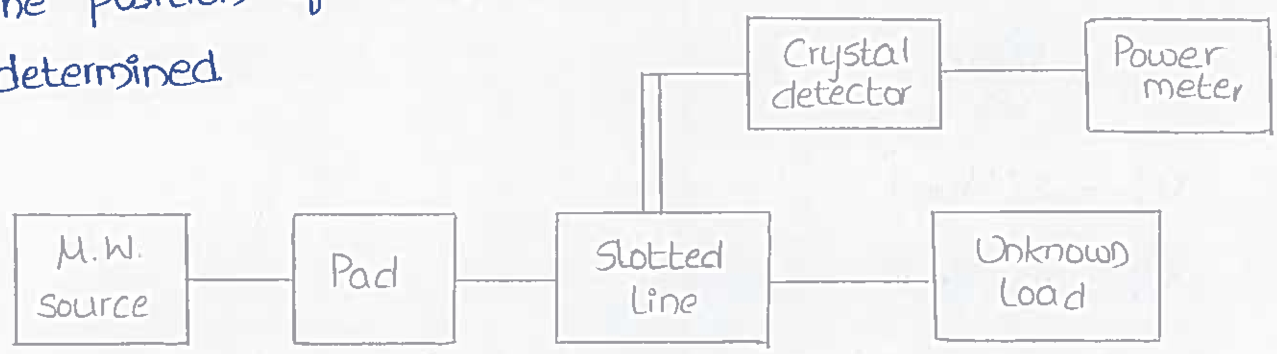
a) using Magic T (as already discussed in Magic T applications)

b) Using slotted line and

c) Using reflectometer.

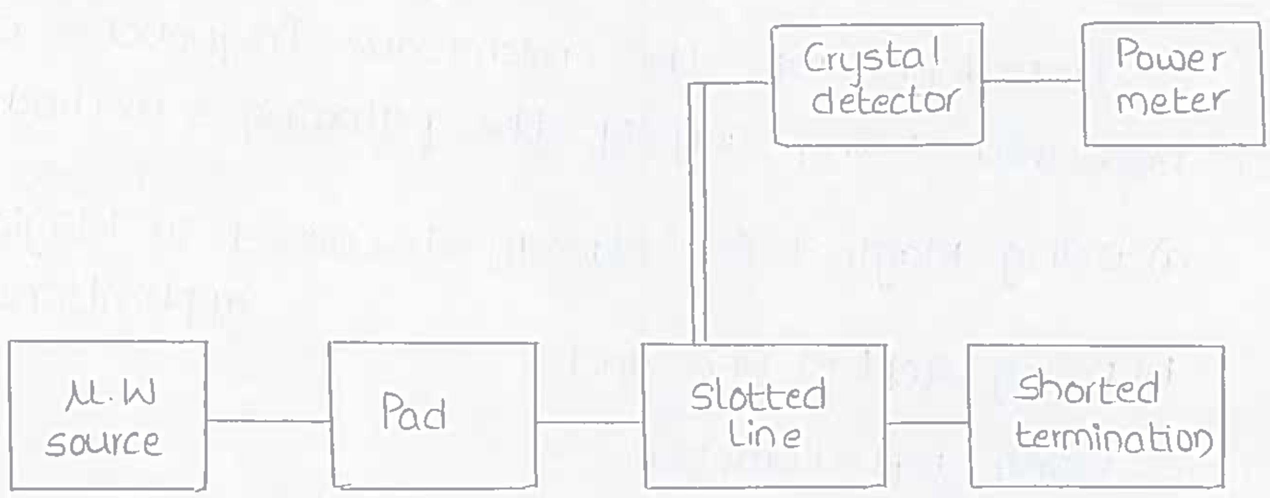
b) Measurement of Impedance using slotted line

Incident and reflected waves will be present proportional to the mismatch of the load under test resulting in standing waves. Using slotted waveguide and with the load $Z_L$ in the circuit, the position of Vmax and Vmin can be accurately determined.

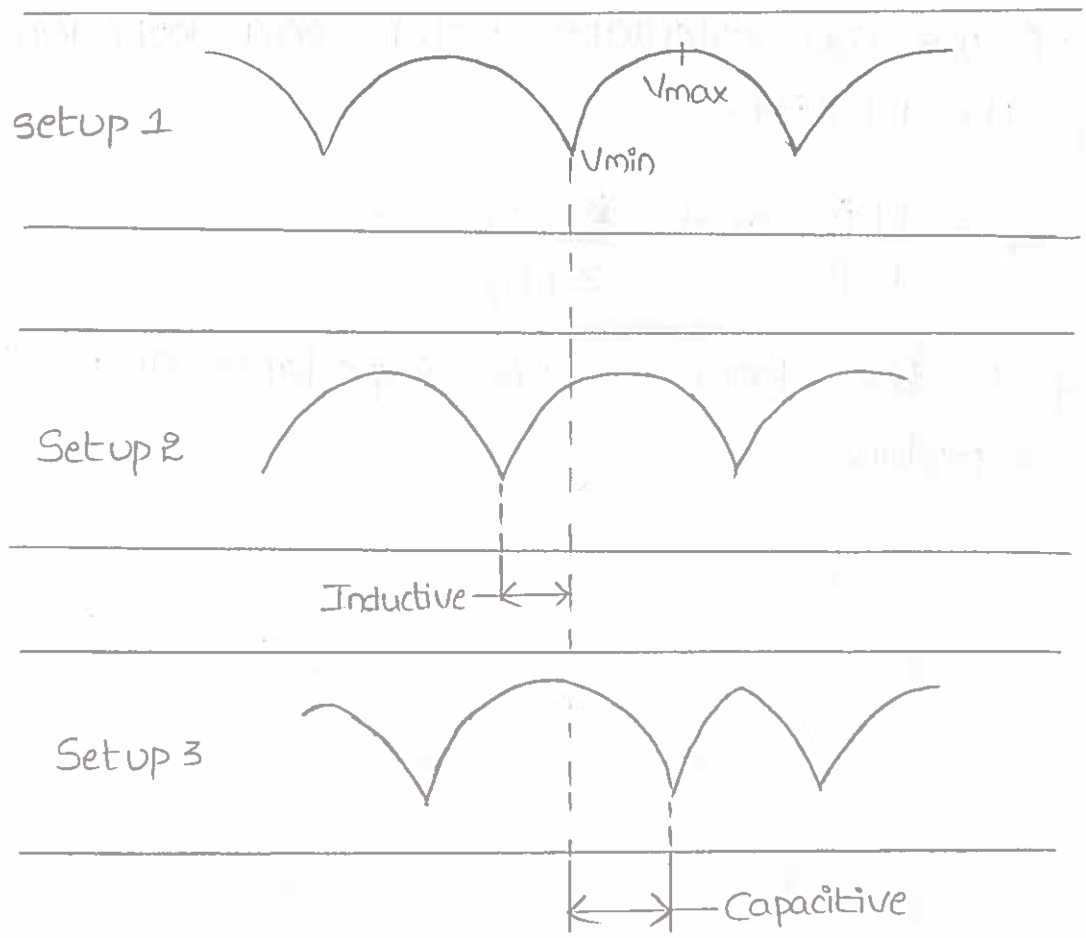



Setup 1, Impedance measurement using slotted line

Now, the load $Z_L$ is replaced by a short circuit and the shift in minimum is measured. If the minimum is shifted to the left, then the impedance is inductive and if it shifts to the right, it is capacitive. Unknown impedance can be obtained by usual methods using the data recorded and a smith chart. Both impedance and reflection coefficient can be obtained in magnitude and phase.




Setup 2, Impedance measurement using slotted line




Output standing waves of setup 1 and 2

c) Measurement of Impedance using Reflectometer

The reflectometer indicates magnitude of impedance but not the phase angle, where as a slotted line gives both. A typical set up for waveguide measurement reflectometer technique is shown. where two directional couplers are used to sample the incident power $P_i$ and the reflected power $P_r$ from load. Both the directional couplers are used to sample the incident) identical. The magnitude of the reflection coefficient, $\rho$ can be directly obtained on the reflectometer from which impedance can be calculated.

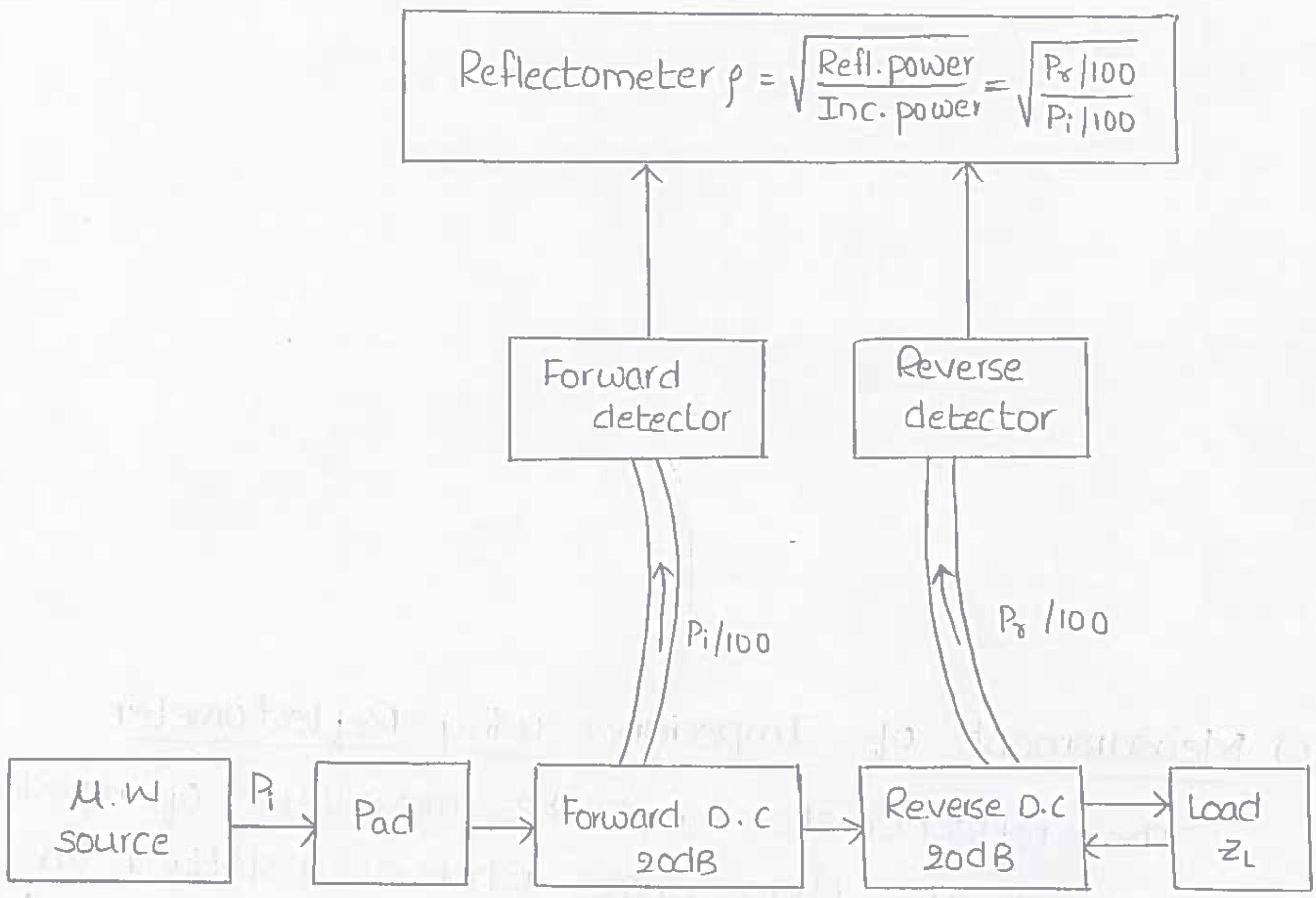
From reflectometer reading we have,

$$\rho = \sqrt{\frac{P_r}{P_i}}$$

knowing $\rho$ we can calculate VSWR and impedance by using the relations.

$$S = \frac{1+\rho}{1-\rho} \quad \text{and} \quad \frac{Z - Z_g}{Z + Z_g} = \rho$$

where $Z_g$ is the known wave impedance and $Z$ is unknown impedance.




Setup for measuring impedance using reflectometer

Due to directional property of the couplers, there will be no interference between forward and reverse waves. The input power is kept to a low level by means of pad. The reflectometer accuracy is greatest at low VSWR.